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On the Commutativity of Prime Rings with One-Sided Ideals and Derivations

Tuna BULAK

Vocational School, Information Security Technology Program,
Doğuş University, Istanbul, Türkiye
e-mail: tbulak@dogus.edu.tr

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Abstract

This article investigates the commutativity of rings using certain properties of derivations on prime rings. Let R ***be a prime ring with center*** $Z(R)$***, and let*** I ***be a nonzero left (right) ideal of*** R***. Suppose that*** d ***is a derivation on*** R ***satisfying the condition*** $d(Z(R)) \neq 0$***. In this paper, we prove that if one of the following conditions holds: (i)*** $d([x,y]) \mp [z,x] \in Z(R)$ ***(ii)*** $d([x,y]) \mp [y,x] \in Z(R)$ ***(iii)*** $d(x \circ y) \mp x \circ z \in Z(R)$ ***(iv)*** $d(x \circ y) \mp z \circ y \in Z(R)$ ***(v)*** $d(xy) \mp [z,x] \in Z(R)$ ***(vi)*** $d(yx) \mp z \circ y \in Z(R)$ ***for all*** $x, y \in I$***, then*** R ***is commutative. Moreover, we provide an example showing that*** R ***must be a prime ring.***

Keywords: *Ring, Prime Ring, Derivation, Commutativity, On Sided Ideal.*

1 Introduction

Throughout this paper, unless otherwise mentioned, R will be a prime ring with center $Z(R)$. For each $a, b \in R$, $[a,b] := ab - ba$ is defined as the commutator and $a \circ b := ab + ba$ is defined as the anti-commutator. Moreover, we will use without explicit mention the following basic identities:

$$[a,a] = 0$$

$$[ab,c] = a[b,c] + [a,c]b$$

$$[a,bc] = b[a,c] + [a,b]c$$

$$[a, b + c] = [a, b] + [a, c]$$
$$[a + b, c] = [a, c] + [b, c]$$

and

$$a \circ (bc) = (a \circ b)c - b[a, c] = b(a \circ z) + [a, b]c$$
$$(ab) \circ c = a(b \circ c) - [a, c]b = (a \circ c)b + a[b, c]$$

For any $a, b \in R$, if $aRb = (0)$ implies $a = 0$ or $b = 0$, then R is said to be a prime ring. By a derivation on R, we mean an additive mapping $d : R \to R$ such that $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Many results in literature show that the structure of a ring R is often closely connected to the behavior of additive mappings defined on R. The first result in this direction is the classical Posner's second theorem. In [1], it is shown that if a prime ring R admits a nonzero derivation d which is centralizing on R, then R is commutative. Recently, many authors have investigated the relationship between certain special types of derivations for ring commutativity, a few of which can be seen as motivations (see, for example, [2], [3], [4], and [5], for which further references can be found). In [6], Ashraf and Rehman proved that if a prime ring R admits a derivation d satisfying $d(xy) \pm xy \in Z(R)$ for all $x, y \in I$, a nonzero ideal of R, then R must be commutative. Based on the above observations, the commutativity of a prime ring R will be investigated by means of a derivative satisfying any of the following properties. Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq (0)$ condition: (i) $d([x, y]) \mp [z, x] \in Z(R)$ (ii) $d([x, y]) \mp [y, x] \in Z(R)$ (iii) $d(x \circ y) \mp x \circ z \in Z(R)$ (iv) $d(x \circ y) \mp z \circ y \in Z(R)$ (v) $d(xy) \mp [z, x] \in Z(R)$ (vi) $d(yx) \mp z \circ y \in Z(R)$ for all $x, y \in I$, then R is commutative.

2 On the Commutativity of Prime Rings with One-Sided Ideals and Derivations

Proposition 2.1. *A group cannot be written as the union of two proper subgroups. This proposition is called the Brauer Trick.*

Lemma 2.2. *Let R be a prime ring with center $Z(R)$, and let d be a derivation on R. Then, $d(Z(R)) \subseteq Z(R)$.*

Proof. For all $a \in Z(R)$ and $r \in R$, we have $ar = ra$. Hence,

$$d(ar) = d(ra)$$

Since d is a derivation, we get

$$d(a)r + ad(r) = d(r)a + rd(a)$$

Here, since $ad(r) = d(r)a$ for $a \in Z(R)$, we conclude

$$d(a)r = rd(a)$$

Thus,

$$d(a)r = rd(a) \Rightarrow [d(a), r] = 0$$

Since $[d(a), r] = 0$, equality will hold for all $r \in R$, and we obtain

$$d(a) \in Z(R)$$

If we arrange this expression for all $a \in Z(R)$, we get

$$d(Z(R)) \subseteq Z(R)$$

□

Lemma 2.3. [7] *Let R be a prime ring with center $Z(R)$. If $a, ab \in Z(R)$, for some $a, b \in R$, then either $a = 0$ or $b \in Z(R)$.*

Lemma 2.4. [8] *Let R be a prime ring. If R contains a nonzero commutative right (left) ideal, then R is commutative.*

Theorem 2.5. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq 0$ condition. If $d([x, y]) \mp [z, x] \in Z(R)$ for all $x, y, z \in I$, then R is commutative.*

Proof. Suppose that I is a right ideal. Using the fact that $d(Z(R)) \neq (0)$, there exists some element $c \in Z(R)$ such that $d(c) \neq 0$. Therefore, it follows from Lemma 2.2 that $d(c) \in Z(R)$. By the hypothesis, we have

$$d([x, y]) - [z, x] \in Z(R), \text{ for all } x, y, z \in I \tag{1}$$

Replacing x by xc in (1), we get that d is a derivation and the commutator property is used:

$$d([xc, y]) - [z, xc] \in Z(R)$$
$$d(x[c, y] + [x, y]c) - x[z, c] - [z, x]c \in Z(R)$$
$$d([x, y]c) - [z, x]c \in Z(R)$$

is obtained, and this expression is rearranged using d being a derivation,

$$d([x, y])c + [x, y]d(c) - [z, x]c \in Z(R), \text{ for all } x, y, z \in I, c \in Z(R)$$

The first and third terms are placed in parentheses with c:

$$(d([x, y]) - [z, x])c + [x, y]d(c) \in Z(R)$$

From this expression, using Lemma 2.2 and $c \in Z(R)$, we obtain:

$$[x, y]d(c) \in Z(R) \tag{2}$$

Here, since $[x, y]d(c) \in Z(R)$ and $d(c) \in Z(R)$, from Lemma 2.2, we get that for all $c \in Z(R)$, $d(c) = 0$ or for all $x, y \in I$, $[x, y] \in Z(R)$. From the hypothesis that $d(Z(R)) \neq (0)$, we obtain:

$$[x, y] \in Z(R), \text{ for all } x, y \in I \tag{3}$$

From expression (3), for all $x, y \in I$ and for all $r \in R$, we get:

$$[r, [x, y]] = 0$$

Here, replacing x with yx in the equality and using the commutator property:

$$0 = [r, [yx, y]] = [r, (y[x, y] + [y, y]x)] = [r, y[x, y]]$$

Rearranging this equality using the commutator property and the hypothesis, we get the equality:

$$0 = [r, y[x, y]] = [r, y][x, y] + y[r, [x, y]] = [r, y][x, y]$$

$$[r, y][x, y] = 0, \text{ for all } x, y \in I \text{ and for all } r \in R$$

In this equality, replacing r with xr and applying the hypothesis, we get:

$$0 = [xr, y][x, y] = x[r, y][x, y] + [x, y]r[x, y] = [x, y]r[x, y]$$

Using the fact that R is a prime ring, for all $x, y \in I$, we get:

$$[x, y]R[x, y] = (0) \Rightarrow [x, y] = (0)$$

That is,

$$[I, I] = (0)$$

Thus, I is commutative. By Lemma 2.4, R is commutative. On the other hand, suppose that

$$d([x, y]) + [z, x] \in Z(R), \text{ for all } x, y, z \in I \tag{4}$$

The derivation d is replaced by $-d$:

$$(-d)([x, y]) + [z, x] \in Z(R), \text{ for all } x, y, z \in I$$

If the necessary arrangements are made from here:

$$-d([x, y]) + [z, x] \in Z(R), \text{ for all } x, y, z \in I$$

As a result, the center is the lower ring:

$$-(-d([x,y]) + [z,x]) \in Z(R), \text{ for all } x,y,z \in I$$

This happens:

$$d([x,y]) - [z,x] \in Z(R), \text{ for all } x,y,z \in I$$

Since the expression obtained is equal to the expression in (1), the prime ring R is commutative due to the proof above. □

Theorem 2.6. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq 0$ condition. If $d([x,y]) \mp [y,x] \in Z(R)$ for all $x,y \in I$, then R is commutative.*

Proof. Suppose that I is a right ideal. Using the fact that $d(Z(R)) \neq (0)$, there exists some element $c \in Z(R)$ such that $d(c) \neq 0$. Therefore, it follows from Lemma 2.2 that $d(c) \in Z(R)$. By the hypothesis, we have

$$d([x,y]) - [y,x] \in Z(R), \text{ for all } x,y \in I \tag{5}$$

Replacing y by yc in (5), we get that d is a derivation and the commutator property is used:

$$d([x,yc]) - [yc,x] \in Z(R)$$
$$d(y[x,c] + [x,y]c) - [y,x]c - y[c,x] \in Z(R)$$
$$d([x,y]c) - [y,x]c \in Z(R)$$

is obtained, and this expression is rearranged using d being a derivation:

$$d([x,y])c + [x,y]d(c) - [y,x]c \in Z(R), \text{ for all } x,y \in I, c \in Z(R)$$

$$(d([x,y]) - [y,x])c + [x,y]d(c) \in Z(R)$$

is found. This expression is used with (5) and $c \in Z(R)$:

$$[x,y]d(c) \in Z(R)$$

is obtained. Since this expression is expression (2) in Theorem 2.5, the rest of the proof is the same. □

Theorem 2.7. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq 0$ condition. If $d(x \circ y) \mp x \circ z \in Z(R)$ for all $x,y,z \in I$, then R is commutative.*

PROOF. Suppose that I is a left ideal. Using the fact that $d(Z(R)) \neq (0)$, there exists some element $c \in Z(R)$ such that $d(c) \neq 0$. Therefore, it follows from Lemma 2.2 that $d(c) \in Z(R)$. By the hypothesis, we have

$$d(x \circ y) - x \circ z \in Z(R), \text{ for all } x, y, z \in I \tag{6}$$

Replacing x by xc in (6), we get:

$$d((xc) \circ y) - (xc) \circ z \in Z(R), \text{ for all } x, y, z \in I$$

Using the commutator and anti-commutator properties,

$$d((x \circ y)c + x[c, y]) - (x \circ z)c + x[c, z] \in Z(R)$$

$$d((x \circ y)c) - (x \circ z)c \in Z(R)$$

is obtained. Since d is a derivation, then

$$d(x \circ y)c + (x \circ y)d(c) - (x \circ z)c \in Z(R)$$

Rearranging terms,

$$(d(x \circ y) - (x \circ z))c + (x \circ y)d(c) \in Z(R)$$

Using (6) and $c \in Z(R)$,

$$(x \circ y)d(c) \in Z(R), \text{ for all } x, y \in I \text{ and for all } d(c) \in Z(R) \tag{7}$$

This implies that since $(x \circ y)d(c) \in Z(R)$ and $d(c) \in Z(R)$ from Lemma 2.3, we get that for all $c \in Z(R)$, $d(c) = 0$ or for all $x, y \in I$, $(x \circ y) \in Z(R)$. From the hypothesis that $d(Z(R)) \neq (0)$,

$$(x \circ y) \in Z(R), \text{ for all } x, y \in I \tag{8}$$

is obtained. Using (8), for all $x, y \in I$ and for all $r \in R$,

$$[(x \circ y), r] = 0$$

is obtained. Replacing y with xy and using the anti-commutator property,

$$0 = [(x \circ (xy)), r] = [x(x \circ y) - [x, x]y), r] = [x(x \circ y), r] = [x, r](x \circ y) + x[(x \circ y), r])$$

Using the commutator property and the hypothesis,

$$0 = [x, r](x \circ y), \text{ for all } x, y \in I \text{ and for all } r \in R$$

Replacing r with rs, where $x, y \in I$, $r, s \in R$,

$$0 = [x, rs](x \circ y) = [x, r]s(x \circ y) + r[x, s](x \circ y) = [x, r]s(x \circ y)$$

Using the fact that R is a prime ring, we obtain

$$[x,r]R(x\circ y)=(0)\Rightarrow [x,r]=(0) \text{ or } (x\circ y)=(0)$$

Defining

$$I_1=\{x\in I:[x,r]=0, r\in R\} \text{ and } I_2=\{x\in I:(x\circ y)=0, y\in I\}$$

Since $I=I_1\cup I_2$ by Proposition 2.1, we have either $I_1=I$ or $I_2=I$. Let's assume that $I_1=I$,

$$[x,r]=0, \text{ for all } x\in I \text{ and for all } r\in R \tag{9}$$

Using (9) and replacing r with ry,

$$0=[x,ry]=r[x,y]+[x,r]y, \text{ for all } x,y\in I \text{ and for all } r\in R$$

Using (9), we get $0=r[x,y]$. Since R is a prime ring, it follows that

$$0=[x,y], \text{ for all } x,y\in I$$

Thus, by Lemma 2.4, R is commutative. Let's assume that $I_2=I$,

$$x\circ y=0, \text{ for all } x,y\in I \tag{10}$$

In this equation, if x is replaced with xz for $z\in I$ and the anti-commutator property is applied,

$$0=(xz)\circ y=(x\circ y)z+x[z,y]$$

From equation (10), we get:

$$0=x[z,y], \text{ for all } x,y,z\in I$$

Replacing z with rz in the obtained expression and applying the above hypothesis, then for all $x,y,z\in I$ and for all $r\in R$,

$$0=x[rz,y]=x(r[z,y]+[r,y]z)=xr[z,y]+x[r,y]z=xr[z,y]$$

is obtained. Here, for all $x,y,z\in I$ and for all $r\in R$,

$$xR[z,y]=(0)$$

Since I is a nonzero left ideal,

$$[z,y]=0, \text{ for all } y,z\in I$$

is found. By Lemma 2.4, the prime ring R is commutative. Similarly, if the same operations are performed for I as a right ideal, it follows that the ring R is commutative.

On the other hand, suppose that for $x, y, z \in I$,

$$d(x \circ y) + x \circ z \in Z(R), \text{ for all } x, y, z \in I \tag{11}$$

The derivation d is replaced by $-d$,

$$(-d)(x \circ y) + x \circ z \in Z(R), \text{ for all } x, y, z \in I$$

is obtained. If the necessary arrangements are made from here,

$$-d(x \circ y) + x \circ z \in Z(R), \text{ for all } x, y, z \in I$$

As a result, the center is the lower ring:

$$-(-d(x \circ y) + x \circ z) \in Z(R), \text{ for all } x, y, z \in I$$

This happens,

$$d(x \circ y) - x \circ z \in Z(R), \text{ for all } x, y, z \in I$$

Since the expression obtained is equal to equation (6), the prime ring R is commutative due to the proof above.

□

Theorem 2.8. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq 0$ condition. If $d(x \circ y) \mp z \circ y \in Z(R)$ for all $x, y, z \in I$, then R is commutative.*

Proof. Suppose that I is a left ideal. Using the fact that $d(Z(R)) \neq (0)$, there exists some element $c \in Z(R)$ such that $d(c) \neq 0$. Therefore, it follows from Lemma 2.2 that $d(c) \in Z(R)$. By the hypothesis, we have

$$d(x \circ y) - z \circ y \in Z(R), \text{ for all } x, y, z \in I \tag{12}$$

Replacing y by cy in (12), we get

$$d(x \circ (cy)) - z \circ (cy) \in Z(R), \text{ for all } x, y, z \in I$$

Using the commutator and anti-commutator properties,

$$d(c(x \circ y) + [x, c]y) - c(z \circ y) + [z, c]y \in Z(R)$$

$$d((x \circ y)c) - (z \circ y)c \in Z(R)$$

is obtained. Since d is a derivation,

$$d(x\circ y)c+(x\circ y)d(c)-(z\circ y)c\in Z(R)$$

Rearranging the first and third terms with parentheses around c,

$$(d(x\circ y)-(z\circ y))c+(x\circ y)d(c)\in Z(R)$$

is found. Using this expression with (12) and $c\in Z(R)$, we obtain $(x\circ y)d(c)\in Z(R),$ for all $x,y\in I$ and for all $d(c)\in Z(R)$

Since this expression corresponds to equation (7) in Theorem 2.7, the rest of the proof is the same. □

Theorem 2.9. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R))\neq 0$ condition. If $d(xy)\mp[z,x]\in Z(R)$ for all $x,y,z\in I$, then R is commutative.*

Proof. Suppose that I is a right ideal. Using the fact that $d(Z(R))\neq(0)$, there exists some element $c\in Z(R)$ such that $d(c)\neq 0$. Therefore, it follows from Lemma 2.2 that $d(c)\in Z(R)$. By the hypothesis, we have

$$d(xy)-[z,x]\in Z(R),\ \text{for all } x,y,z\in I \tag{13}$$

Since d is a derivation, we rearrange using the commutator property,

$$d(x)y+xd(y)-[z,x]\in Z(R),\ \text{for all } x,y,z\in I \tag{14}$$

Replacing x by xc in (14), we get

$$d(xc)y+xcd(y)-[z,xc]\in Z(R),\ \text{for all } x,y,z\in I$$

Since d is a derivation, we use the commutator property,

$$(d(x)c+xd(c))y+xcd(y)-[z,x]c-x[z,c]\in Z(R),\ \text{for all } x,y,z\in I$$

$$d(x)cy+xd(c)y+xcd(y)-[z,x]c\in Z(R),\ \text{for all } x,y,z\in I$$

Grouping the 1st, 3rd, and 4th terms in parentheses with c,

$$(d(x)y+xd(y)-[z,x])c+xyd(c)\in Z(R),\ \text{for all } x,y,z\in I$$

Using this expression with (14) and $c\in Z(R)$, we obtain

$$xyd(c)\in Z(R),\ \text{for all } x,y\in I \tag{15}$$

Similarly, if we swap x and y in the hypothesis, then for all $x,y\in I$ and for all $c\in Z(R)$,

$$yxd(c)\in Z(R),\ \text{for all } x,y\in I \tag{16}$$

Subtracting (16) from (15), we obtain

$$[x,y]d(c) \in Z(R), \text{ for all } x,y \in I, c \in Z(R)$$

Since this expression corresponds to equation (2) in Theorem 2.5, the rest of the proof is the same. □

Theorem 2.10. *Let R be a prime ring with center $Z(R)$, and let I be a nonzero left (right) ideal of R. Suppose that d is a derivation on R satisfying the $d(Z(R)) \neq 0$ condition. If $d(yx) \mp z \circ y \in Z(R)$ for all $x, y, z \in I$, then R is commutative.*

Proof. Suppose that I is a right ideal. Using the fact that $d(Z(R)) \neq (0)$, there exists some element $c \in Z(R)$ such that $d(c) \neq 0$. Therefore, it follows from Lemma 2.2 that $d(c) \in Z(R)$. By the hypothesis, we have

$$d(yx) - z \circ y \in Z(R), \text{ for all } x,y,z \in I \tag{17}$$

Since d is a derivation, we rearrange using the anti-commutator property,

$$d(y)x + yd(x) - z \circ y \in Z(R), \text{ for all } x,y,z \in I \tag{18}$$

Replacing y by cy in (18), we get

$$d(yc)x + ycd(x) - z \circ (cy) \in Z(R), \text{ for all } x,y,z \in I$$

Since d is a derivation, we use the anti-commutator property,

$$(d(y)c + yd(c))x + ycd(x) - c(z \circ y) + [z,c]y \in Z(R), \text{ for all } x,y,z \in I$$

$$d(y)cx + yd(c)x + ycd(x) - c(z \circ y) \in Z(R), \text{ for all } x,y,z \in I$$

Grouping the 1st, 3rd, and 4th terms in parentheses with c,

$$(d(y)x + yd(x) - (z \circ y))c + yxd(c) \in Z(R), \text{ for all } x,y,z \in I$$

Using this expression with (18) and $c \in Z(R)$, we obtain

$$yxd(c) \in Z(R), \text{ for all } x,y \in I$$

Since expression (16) is expression. Similarly, if we swap x and y in the hypothesis, then for all $x, y \in I$ and for all $c \in Z(R)$,

$$xyd(c) \in Z(R), \text{ for all } x,y \in I$$

Since expression (15) is obtained. Now, if expression Subtracting (16) from (15), we obtain

$$[x,y]d(c) \in Z(R), \text{ for all } x,y \in I, c \in Z(R)$$

Since this expression corresponds to equation (2) in Theorem 2.5, the rest of the proof is the same. □

Example 2.11. *Let Q be the set of all integers, and define the ring*

$$R = \left\{ \begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} : x, y, z \in \mathbb{Q} \right\}.$$

Define the ideal

$$I = \left\{ \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} : x, z \in \mathbb{Q} \right\}.$$

It is clear that R is a ring and I is an ideal of R. Define the mapping $d : R \to R$ by

$$d\left(\begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & 0 & x - z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, it is easy to check that d is a nonzero derivative on R, and hence $d(Z(R)) \neq (0)$. Moreover,

$$\begin{array}{ll} (i) & d([x,y]) \mp [z,x] \in Z(R) \\ (ii) & d([x,y]) \mp [y,x] \in Z(R) \\ (iii) & d(x \circ y) \mp x \circ z \in Z(R) \\ (iv) & d(x \circ y) \mp z \circ y \in Z(R) \\ (v) & d(xy) \mp [z,x] \in Z(R) \\ (vi) & d(yx) \mp z \circ y \in Z(R) \end{array}$$

for all $x, y, z \in I$. However, since R is not a prime ring, R is non-commutative. In other words, the condition of primeness in theorems is not superfluous.

3 Conclusion

This study establishes the commutativity of a prime ring R under the conditions specified in six theorems, using a proposition and three lemmas, one of which is original. Additionally, a counterexample is provided, demonstrating that even when a derivation d exists in a non-prime ring R and the conditions of the theorems are satisfied, R is not necessarily commutative. This result highlights the necessity of the restrictions imposed on the hypotheses of the presented theorems.

4 Open Problem

In the future, researchers may explore commutativity using transformations such as derivations, reverse derivations, generalized derivations, and generalized reverse derivations under different conditions on prime or semiprime rings.

They could use closed Lie ideals, prime ideals, or semiprime ideals instead of one-sided ideals in these investigations.

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