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Short Introduction to SuperHyperGraph Theory with Some Applications

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Abstract

A finite* hypergraph *extends an ordinary graph by permitting each* hyperedge *to connect an arbitrary nonempty subset of vertices, thereby encoding genuinely multiway interactions. Building on this idea, a finite* SuperHyperGraph *is formed by iterating the powerset construction, so that set-valued objects created at one level can act as vertices (and hence as potential edge endpoints) at the next. This provides a principled framework for representing hierarchical, nested, and multilayer relational structures. In this paper, we introduce SuperHyperGraphs and several related concepts, illustrating them with concrete examples.

Keywords: *Recursive SuperHyperGraph, HyperGraph, SuperHyperGraph, Hierarchical SuperHyperGraph.*

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1 SuperHyperGraphs

We first review the set-theoretic constructions underlying SuperHyperGraphs and then recall the associated graph-theoretic terminology [1, 2, 3, 4].

Definition 1.1 (Hypergraph) *[5, 6] A* hypergraph *is an ordered pair $H = (V, E)$ where*

- *V is a finite set of* vertices*, and*

- *E is a finite family of nonempty subsets of V, called* hyperedges*.*

Thus a hyperedge may involve more than two vertices, allowing one to encode genuinely multiway relations.

Definition 1.2 (n-fold iterated powerset) *[7, 8] Let X be a set. Define $\mathcal{P}^1(X) := \mathcal{P}(X)$ and, for $n \geq 1$, recursively set*

$$\mathcal{P}^{n+1}(X) := \mathcal{P}\big(\mathcal{P}^n(X)\big).$$

When the empty set is excluded, we write

$$\mathcal{P}^n_*(X) := \mathcal{P}^n(X) \setminus \{\emptyset\}.$$

Definition 1.3 (n-SuperHyperGraph) *(see [1]) Let V_0 be a finite, nonempty base set, and define iterated powersets by*

$$\mathcal{P}^0(V_0) := V_0,$$

$$\mathcal{P}^{k+1}(V_0) := \mathcal{P}\big(\mathcal{P}^k(V_0)\big) \quad (k \in \mathbb{N} \cup \{0\}).$$

*For $n \geq 0$, an n-*SuperHyperGraph on *V_0 is a pair*

$$\mathrm{SHG}^{(n)} = (V, E)$$

such that

$$V \subseteq \mathcal{P}^n(V_0) \qquad \text{and} \qquad E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

*The elements of V are called n-*supervertices*, and the elements of E are called* superhyperedges*; equivalently, each superhyperedge is a nonempty subset of the supervertex set V.*

As a brief comparison, Table 1 summarizes the main differences between a Hypergraph and an n-SuperHyperGraph. Compared with a Hypergraph, an n-SuperHyperGraph captures nested, hierarchical, and multi-level relations by allowing set-based vertices, thereby modeling richer higher-order structures and interactions in practice. In fact, SuperHyperGraphs have recently been discussed in a wide variety of publications [9, 10, 11, 12, 13].

A concrete example of a SuperHyperGraph is given below.

Example 1.4 (Multi-level product bundles in an e-commerce platform) *Consider an e-commerce platform that offers individual products, simple bundles, and higher-level bundle packages.*

Base set. Let

$$V_0 = \{\text{Laptop}, \text{Mouse}, \text{Dock}, \text{Monitor}\}.$$

Table 1: Comparison of Hypergraph and n-SuperHyperGraph

Aspect	**Hypergraph**	**n-SuperHyperGraph**
Basic form	$H = (V, E)$	$\mathrm{SHG}^{(n)} = (V, E)$
Vertex set	V is a finite set of ordinary vertices	$V \subseteq \mathcal{P}^n(V_0)$, so vertices are n-th level set-based objects
Edge set	E is a family of nonempty subsets of V	$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so each superhyperedge is a nonempty subset of the supervertex set
Structural level	Single-level structure	Multi-level or hierarchical structure determined by the iterated powerset level n
Underlying objects	Vertices are atomic objects	Supervertices may themselves encode nested collections of lower-level objects
Main purpose	Models multiway relations among ordinary vertices	Models multiway relations among higher-order, nested, or hierarchical entities
Set-theoretic basis	Uses only subsets of the vertex set	Uses iterated powersets of a base set V_0
Special case	Standard model	For $n = 0$, one obtains the ordinary hypergraph-type setting on a subset of V_0

Level-1 bundles. Define

$$b_1 := \{\text{Laptop}, \text{Mouse}\}, \qquad b_2 := \{\text{Dock}\},$$

$$b_3 := \{\text{Laptop}, \text{Dock}\}, \qquad b_4 := \{\text{Monitor}\}.$$

Choose $n = 2$. A 2-supervertex is a set of bundles. Define

$$v_1 := \{b_1, b_2\}, \qquad v_2 := \{b_3, b_4\}.$$

Here, v_1 represents a basic work package, while v_2 represents an extended desk package.

Let

$$V := \{v_1, v_2\} \subseteq \mathcal{P}^2(V_0).$$

Superhyperedge. Define

$$E := \{\{v_1, v_2\}\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

This superhyperedge models a promotion that jointly recommends both higher-level packages.

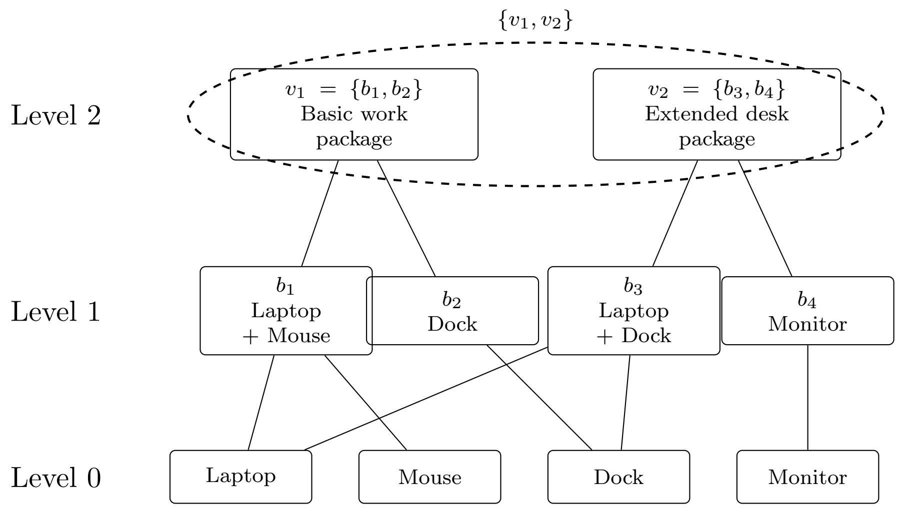

Figure 1: A simple 2-SuperHyperGraph for multi-level product bundles in an e-commerce platform.

Thus

$$\mathrm{SHG}^{(2)} = (V, E)$$

is a concrete 2*-SuperHyperGraph describing multi-level product bundling in e-commerce.*

2 (m, n)-SuperHyperGraph

An (m, n)-SuperHyperGraph encodes multiway relations among supervertices modeled as maps $\mathcal{P}^m(S) \to \mathcal{P}^n(S)$; its edges are nonempty collections of such maps [2].

Definition 2.1 ((m,n)-SuperHyperGraph) *Fix $m, n \in \mathbb{N}$ and a nonempty base set S. Let*

$$\mathfrak{F}_{m,n}(S) := \{\, f : \mathcal{P}^m(S) \to \mathcal{P}^n(S) \,\}$$

*be the class of all (m, n)-superhyperfunctions on S, where $\mathcal{P}^t$ denotes the t-fold iterated powerset. An (m, n)-*SuperHyperGraph *over S is a triple*

$$\mathrm{SHG}^{(m,n)} = (V, \mathcal{E}, \partial),$$

where:

(i) $V \subseteq \mathfrak{F}_{m,n}(S)$ *is a nonempty set (its elements are called* (m,n)-supervertices*);*

(ii) $\mathcal{E}$ *is a nonempty set of (super)edge identifiers;*

(iii) $\partial : \mathcal{E} \to \mathcal{P}^*(V)$ *is an incidence map, where* $\mathcal{P}^*(V) := \mathcal{P}(V) \setminus \{\emptyset\}$.

For $e \in \mathcal{E}$*, the set* $\partial(e) \subseteq V$ *is called the* incidence set *(or endpoint set) of the superhyperedge* e.

Example 2.2 (Emergency-department protocol coordination) *Consider an emergency department in which clinicians combine multiple clinical protocols to decide* bundles *of actions from observed findings.*
Base set. Let the ground set S *contain atomic findings and atomic actions:*

$$S = \{\mathrm{F}, \mathrm{O}, \mathrm{C}, \mathrm{B}, \mathrm{T}, \mathrm{X}, \mathrm{K}\},$$

where F *= fever,* O *= low oxygen saturation,* C *= chest pain,* B *= order blood test,* T *= order CT imaging,* X *= start oxygen therapy,* K *= call cardiology.*
Choose $(m,n) = (1,2)$*. Then a supervertex is a function*

$$f : \mathcal{P}(S) \longrightarrow \mathcal{P}^2(S) = \mathcal{P}(\mathcal{P}(S)),$$

mapping a set of observed findings (and any already-selected items) to a family of action-bundles *(each bundle is a subset of* S*).*
Supervertices (protocols). Define three $(1,2)$*-superhyperfunctions:*

$$f_{\mathrm{resp}}(A) := \begin{cases} \{\{\mathrm{X},\mathrm{B}\},\{\mathrm{X},\mathrm{T}\}\}, & \text{if } \mathrm{O} \in A, \\ \emptyset, & \text{otherwise}, \end{cases}$$

$$f_{\mathrm{card}}(A) := \begin{cases} \{\{\mathrm{K},\mathrm{B}\},\{\mathrm{K},\mathrm{T}\}\}, & \text{if } \mathrm{C} \in A, \\ \emptyset, & \text{otherwise}, \end{cases}$$

$$f_{\mathrm{inf}}(A) := \begin{cases} \{\{\mathrm{B}\},\{\mathrm{B},\mathrm{X}\}\}, & \text{if } \mathrm{F} \in A, \\ \emptyset, & \text{otherwise}. \end{cases}$$

Let

$$V := \{f_{\mathrm{resp}},\ f_{\mathrm{card}},\ f_{\mathrm{inf}}\} \subseteq \mathfrak{F}_{1,2}(S).$$

Superhyperedges (joint-use situations). Let the edge-identifier set be

$$\mathcal{E} := \{e_{\mathrm{pne}},\ e_{\mathrm{acs}},\ e_{\mathrm{complex}}\},$$

with incidence map $\partial : \mathcal{E} \to \mathcal{P}^*(V)$ *defined by*

$$\partial(e_{\mathrm{pne}}) = \{f_{\mathrm{resp}}, f_{\mathrm{inf}}\}, \qquad \partial(e_{\mathrm{acs}}) = \{f_{\mathrm{resp}}, f_{\mathrm{card}}\}, \qquad \partial(e_{\mathrm{complex}}) = \{f_{\mathrm{resp}}, f_{\mathrm{card}}, f_{\mathrm{inf}}\}.$$

Interpretation: each superhyperedge collects the protocols that must be considered together in a given clinical scenario (e.g., pneumonia-like, acute-coronary-like, or a complex case). Thus $\mathrm{SHG}^{(1,2)} = (V, \mathcal{E}, \partial)$ *is a concrete* (m,n)*-SuperHyperGraph arising in real-world decision support.*

3 Hierarchical Undirected SuperHyperGraphs

A hierarchical superhypergraph allows vertices drawn from several iterated-powerset levels and permits *mixed-level* edges, while imposing a coherence requirement that guarantees level-to-level consistency.

Definition 3.1 (Hierarchical SuperHyperGraph of height r) *Let V_0 be a finite, nonempty base set and fix $r \in \mathbb{N}_0$. Define the* nonempty powerset tower $(\mathcal{P}^{\langle k\rangle}(V_0))_{k=0}^{r}$ *by*

$$\mathcal{P}^{\langle 0\rangle}(V_0) \coloneqq V_0, \qquad \mathcal{P}^{\langle k+1\rangle}(V_0) \coloneqq \mathcal{P}\big(\mathcal{P}^{\langle k\rangle}(V_0)\big) \setminus \{\emptyset\} \quad (0 \le k < r).$$

Set the hierarchical universe *to be*

$$\mathcal{U}_r(V_0) \ \coloneqq \ \bigcup_{k=0}^{r} \mathcal{P}^{\langle k\rangle}(V_0).$$

For $x \in \mathcal{U}_r(V_0)$, define its level *by*

$$\ell(x) \ \coloneqq \ \min\{\, k \in \{0,1,\dots,r\} : x \in \mathcal{P}^{\langle k\rangle}(V_0)\,\}.$$

A hierarchical SuperHyperGraph of height r *on V_0 is a pair*

$$\mathbb{H}^{\langle r\rangle} = (V, E)$$

satisfying the following conditions:

(H1) (Hierarchical vertex set). *V is a finite, nonempty set such that*

$$V \subseteq \mathcal{U}_r(V_0).$$

The elements of V are called hierarchical supervertices*.*

(H2) (Cross-level edges). *E is a finite family of nonempty subsets of V, that is,*

$$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

The elements of E are called hierarchical superhyperedges*. In particular, a single edge may contain vertices from different levels.*

(H3) (Coherence / downward closure). *Whenever $X \in V$ has level at least* 1*, all of its immediate constituents are also required to be vertices:*

$$X \subseteq V.$$

For each $k \in \{0,\dots,r\}$, the k-th layer *is defined by*

$$V_k \ \coloneqq \ \{\, x \in V : \ \ell(x) = k \,\}, \qquad \text{so that} \qquad V = \dot{\bigcup}_{k=0}^{r} V_k.$$

Example 3.2 (Hierarchical sensor monitoring as a height-2 hierarchical SuperHyperG
Consider a building-wide monitoring system.

Ground level (V_0). Let the base set be the sensors installed in the building:

$$V_0 = \{\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3, \mathrm{S}_4\},$$

where each S_i is a physical sensor node.

Height $r = 2$ (three levels: $0, 1, 2$). Level-0 vertices represent individual sensors. Level-1 vertices represent nonempty sensor clusters *(e.g., devices within a room or corridor segment). Level-2 vertices represent* nonempty collections of clusters *(e.g., zones composed of multiple rooms).*

Hierarchical supervertex set. Define the following mixed-level vertices:

$$V_0' := \{\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3, \mathrm{S}_4\},$$

$$c_1 := \{\mathrm{S}_1, \mathrm{S}_2\}, \quad c_2 := \{\mathrm{S}_2, \mathrm{S}_3\}, \quad c_3 := \{\mathrm{S}_3, \mathrm{S}_4\},$$
$$z_1 := \{c_1, c_2\}, \qquad z_2 := \{c_2, c_3\}.$$

Let
$$V := V_0' \cup \{c_1, c_2, c_3\} \cup \{z_1, z_2\} \subseteq \mathcal{U}_2(V_0).$$

Interpretation: c_i are local clusters of nearby sensors, while z_1, z_2 are higher-level zones formed from clusters.

Downward-closure (coherence). Since $c_1, c_2, c_3 \in V$ have level 1, their constituents S_i are also in V. Since $z_1, z_2 \in V$ have level 2, their constituents c_i are also in V. Hence the coherence requirement $X \subseteq V$ for all $X \in V$ with $\ell(X) \geq 1$ is satisfied.

Cross-level edges. Define a set of hierarchical superhyperedges by

$$E := \big\{\{\mathrm{S}_2, c_2, z_1\},\ \{c_2, c_3, z_2\},\ \{\mathrm{S}_1, c_1\}\big\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Interpretation: the edge $\{\mathrm{S}_2, c_2, z_1\}$ links an individual sensor, its local cluster, and a higher-level zone, capturing an alarm escalation path from device-level signals to cluster-level and zone-level aggregation.

Therefore $\mathbb{H}^{\langle 2 \rangle} = (V, E)$ is a concrete hierarchical SuperHyperGraph of height $r = 2$ arising from real-world multi-level sensor monitoring. Figure 2 illustrates the mixed-level vertices, the containment relations required by downward closure, and the cross-level hierarchical superhyperedges.

4 Recursive HyperGraph

A *Recursive HyperGraph* is a hypergraph in which hyperedges are allowed to contain not only ordinary vertices but also lower-level hyperedges as elements,

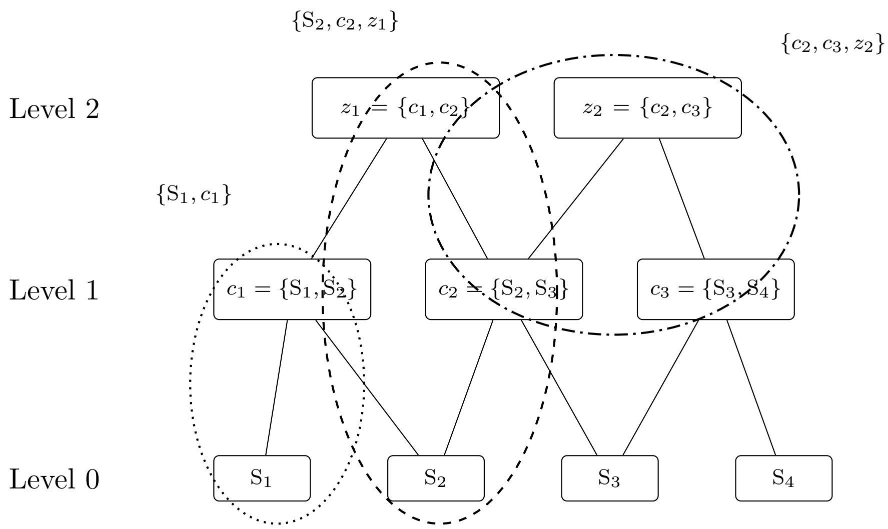

Figure 2: A height-2 hierarchical SuperHyperGraph for sensor monitoring. Straight segments indicate downward-closure/containment relations across levels, while the dotted and dashed closed curves represent cross-level hierarchical superhyperedges.

thereby supporting nested (self-referential) incidence up to a bounded recursion depth [14, 15]. An (n, k)-recursive SuperHyperGraph combines level-n supervertices (via iterated powersets) with depth-k recursive superhyperedges that may contain supervertices and lower-level edges as elements [16, 17].

Definition 4.1 (Depth-k powerset universe) *[14, 15] Let S be a nonempty set and let $k \in \mathbb{N} \cup \{0\}$. Define a hierarchy of sets $(S_i)_{i \geq 0}$ by*

$$S_0 := S, \qquad S_i := \mathcal{P}\Big(\bigcup_{j=0}^{i-1} S_j\Big) \quad (i \geq 1).$$

The depth-k powerset universe *over S is*

$$2_{S,k} := \mathcal{P}\Big(\bigcup_{i=0}^{k} S_i\Big).$$

Definition 4.2 (k-recursive hypergraph) *[14, 15] Let V be a finite vertex set and let $k \in \mathbb{N} \cup \{0\}$. A* k-recursive hypergraph *is a pair*

$$H = (V, E)$$

such that

$$E \subseteq 2_{V,k} \setminus \{\emptyset\},$$

where $2_{V,k}$ is the depth-k powerset universe from Definition 4.1 applied to $S = V$.

In particular, when $k = 0$ one has $2_{V,0} = \mathcal{P}(V)$ and therefore $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so H reduces to an ordinary hypergraph.

Definition 4.3 ((n, k)-recursive SuperHyperGraph) *[17] Fix a base (ground) set V_0 and let $n, k \in \mathbb{N} \cup \{0\}$.*
(Iterated powersets). Define the iterated powersets by

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{n+1}(V_0) = \mathcal{P}\big(\mathcal{P}^n(V_0)\big) \quad (n \geq 0).$$

*A (n, k)-*recursive SuperHyperGraph *is a pair*

$$\mathrm{RSHG}^{(n,k)} = (V, E)$$

satisfying:

- (i) *(Hierarchical supervertex set). $V \subseteq \mathcal{P}^n(V_0)$.*
- (ii) *(Recursive superhyperedge family). $E \subseteq 2_{V,k} \setminus \{\emptyset\}$, where $2_{V,k}$ is the depth-k powerset universe constructed from $S = V$ as in Definition 4.1.*

Example 4.4 (Hierarchical incident response in a cloud service) *Consider a cloud-based microservice system.*
Ground level. Let V_0 be the set of basic components (services and on-call teams), e.g.,

$$V_0 = \{\mathrm{Auth}, \mathrm{API}, \mathrm{DB}, \mathrm{Cache}, \mathrm{SRE}, \mathrm{Sec}\}.$$

Choose $n = 2$ (supervertices as "sets of component-groups"). A level-2 supervertex is an element of $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$, so it represents a collection of component groups. *For instance, define*

$$v_1 := \big\{\{\mathrm{Auth}, \mathrm{API}\}, \{\mathrm{DB}\}\big\}, \qquad v_2 := \big\{\{\mathrm{Cache}\}, \{\mathrm{SRE}\}\big\},$$

$$v_3 := \big\{\{\mathrm{Sec}\}, \{\mathrm{API}, \mathrm{DB}\}\big\}.$$

Interpretation: v_1 encodes the application-and-database dependency group, v_2 encodes the caching layer with the SRE team, and v_3 encodes a security-related incident involving the API/DB boundary.

Let the supervertex set be

$$V := \{v_1, v_2, v_3\} \subseteq \mathcal{P}^2(V_0).$$

Choose $k = 1$ (recursive superhyperedges may include supervertices and lower-level edges). For $k = 1$, the depth-1 universe $2_{V,1}$ contains subsets of V as well *as subsets whose elements may themselves be subsets of V (i.e., "edge-of-edges" structures).*

Define two level-0 incident bundles (ordinary hyperedges on V):

$$e_1 := \{v_1, v_2\} \subseteq V, \qquad e_2 := \{v_2, v_3\} \subseteq V.$$

Interpretation: e_1 models an incident where the application/database group and caching/SRE group must coordinate; e_2 models an incident where caching/SRE and security/API-DB boundary must coordinate.

Now define a recursive *superhyperedge that bundles these incident bundles:*

$$e_3 := \{e_1, e_2\} \in \mathcal{P}\big(\mathcal{P}(V)\big) \subseteq 2_{V,1}.$$

Interpretation: e_3 represents a higher-level escalation (e.g., a major outage) whose resolution requires handling both incident bundles e_1 and e_2 together, capturing a nested coordination structure.

Finally, set

$$E := \{e_1, e_2, e_3\} \subseteq 2_{V,1} \setminus \{\emptyset\}.$$

Then $\mathrm{RSHG}^{(2,1)} = (V, E)$ *is a concrete $(n, k) = (2, 1)$-recursive SuperHyperGraph modeling real-world hierarchical incident management with nested escalation bundles. Figure 3 visualizes the base components, the intermediate component-groups, the level-2 supervertices, the ordinary incident bundles e_1, e_2, and the recursive superhyperedge $e_3 = \{e_1, e_2\}$.*

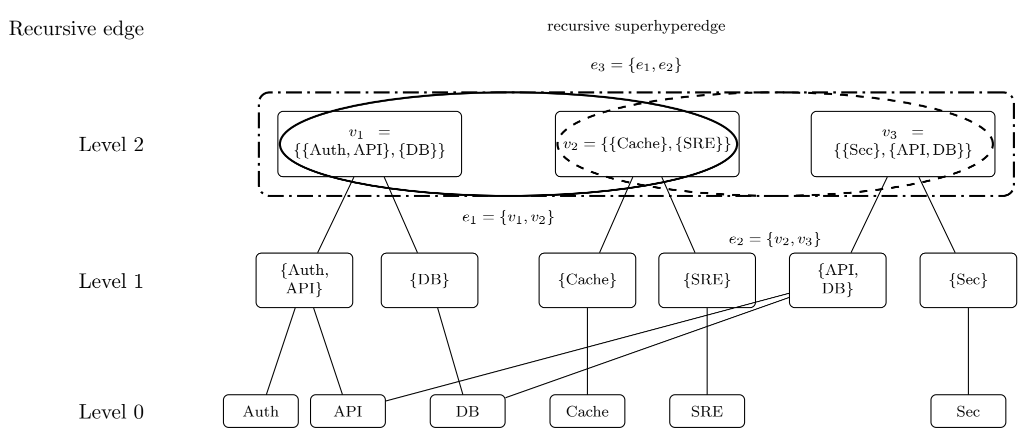

Figure 3: A concrete $(2, 1)$-recursive SuperHyperGraph for hierarchical incident response in a cloud service. Thin segments indicate containment relations across levels, the solid and dashed closed curves represent the ordinary incident bundles e_1 and e_2, and the outer dash-pattern box represents the recursive superhyperedge $e_3 = \{e_1, e_2\}$.

5 Open Problems

The SuperHyperGraph frameworks introduced here raise several natural research questions.

(i) **Algorithmic design.** Can one develop efficient algorithms for SuperHyperGraphs and their variants (e.g., construction, traversal, matching, covering, domination, coloring, optimization, and recognition problems), together with complexity and approximation guarantees?

(ii) **Neural-network applications.** Can SuperHyperGraphs be used as underlying data structures for neural models that learn from hierarchical and multiway relations, for instance by extending message-passing and attention mechanisms to iterated-powerset vertex levels? (cf.[18, 19])

(iii) **Graph parameters and invariants.** Which classical graph/hypergraph parameters admit meaningful analogues in the SuperHyperGraph setting (e.g., degree notions, connectivity, acyclicity, width measures, spectral quantities, domination/covering numbers), and how do these invariants behave under level changes and natural SuperHyperGraph operations?

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Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

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