Mixed SuperHyperStructure and Some Open Problems

T. Fujita

Independent Researcher, Tokyo, Japan.

e-mail:Takaaki.fujita060@gmail.com

Abstract

Hyperstructures extend classical algebraic systems by allowing operations to return sets *of possible results rather than single outputs. Iterating this idea across multiple layers of collections gives rise to* superhyperstructures, *which provide a natural framework for describing hierarchical and multi-level interactions. This work introduces two mixed higher-order frameworks:* **Mixed HyperStructures** *and* **Mixed Super-HyperStructures**. *In addition, I have included several open problems at the end of this paper.*

1 Introduction

1.1 HyperStructure and SuperHyperStructure

In this paper, the term *Structure* is used in a broad sense: it covers both abstract mathematical objects (such as graphs, topological spaces, algebras, automata, or games) and idealized representations of real–world systems. Many of these systems possess an intrinsically hierarchical organization, and for such situations the formalisms of *hyperstructures* and *superhyperstructures* provide a convenient and expressive framework.

A *hyperstructure* relaxes the usual notion of an algebraic structure by allowing a binary operation to return an entire *subset* of possible outcomes instead of a single element (see, e.g., [1, 2, 3, 4]). This point of view has motivated many hyperstructure–based models, including HyperGraphs [5, 6, 7], HyperFuzzy Sets [8, 9], HyperNeutrosophic Sets [10], and HyperAlgebras [11, 12], among others, all of which are closely connected to the hyperstructure paradigm.

By iterating the powerset construction n times one obtains the iterated powerset $\mathrm{PS}_n(H)$, and operations acting on these objects lead to the notion of an *n-superhyperstructure* (see, for example, [13, 14, 1]). Such n-superhyperstructures are intended to model complex, multi–layered phenomena in a transparent way, by organizing interactions across several levels of aggregation. In analogy with hyperstructures, a number of superhyperstructure–based concepts have been proposed, including SuperHyperGraphs [15, 16, 17, 18, 19], SuperHyperFuzzy Sets [20, 21], and SuperHyperAlgebras [22, 23].

For later reference, Table 1 summarizes the basic variants that will appear in this work.

Table 1: Classical structures, hyperstructures, superhyperstructures, and (m, n)-superhyperstructures: underlying domains and typical operation signatures

Structure type	Underlying domain	Typical operation signature
Classical structure	H	$\#_0 : H^m \to H$
Hyperstructure	H	$\circ : H \times H \to \mathrm{PS}(H)$
n-superhyperstructure	$\mathrm{PS}_n(H)$	$\circ : \left(\mathrm{PS}_n(H)\right)^k \to \mathrm{PS}_n(H)$
(m, n)-superhyperstructure	$\mathrm{PS}_m(H)$	$\star : \left(\mathrm{PS}_m(H)\right)^k \to \mathrm{PS}_n(H)$

Notation. For a set H, its powerset is $\mathrm{PS}(H)$. The iterated powersets are defined by $\mathrm{PS}_0(H) = H$ and $\mathrm{PS}_{n+1}(H) = \mathrm{PS}\left(\mathrm{PS}_n(H)\right)$ for $n \geq 0$. We also write $\mathrm{PS}^m(H) = \mathrm{PS}_m(H)$ when no confusion arises. If the empty set is to be excluded, we use $\mathrm{PS}_*^n(H) = \mathrm{PS}_n(H) \setminus \{\emptyset\}$. As usual, H^m denotes the m-fold Cartesian product, and $k \geq 1$ denotes the arity of the operation.

1.2 Our Contributions

In this paper, we introduce and study two mixed higher–order formalisms: Mixed HyperStructures, Mixed SuperHyperStructures. A *Mixed HyperStructure* couples several typed powersets with hyperoperations that map heterogeneous input subsets to output subsets, thereby encoding cross–type relational aggregation within a single framework. A *Mixed SuperHyperStructure* extends this idea to iterated powersets: operations act on nested collections, providing a unified way to describe hierarchical scenarios and multi–level interactions.

Taken together, these constructions offer a flexible language for representing hierarchical and layered phenomena that simultaneously arise across different mathematical and real–world structures. In addition, I have included several open problems at the end of this paper.

2 Preliminaries

In this section we fix notation and recall the basic notions used throughout the paper. Unless explicitly stated otherwise, all sets are assumed to be finite.

2.1 Hyperstructures and SuperHyperStructures

Many mathematical models and real–world systems naturally exhibit several hierarchical layers (for instance, elements, groups of elements, and groups of groups). To describe such multi–level relationships in a uniform way, one may work with the notions of *hyperstructures* and *superhyperstructures.* Informally, a hyperstructure generalizes an algebraic structure by allowing an operation to return a *set* of possible outcomes rather than a single output. Iterating the powerset construction then leads to higher–order domains on which operations can act on *nested* collections.

Definition 2.1 (Powerset). [24] For a set S, its *powerset* is

$$\mathrm{PS}(S) = \{\, A \mid A \subseteq S \,\}.$$

In particular, $\emptyset \in \mathrm{PS}(S)$ and $S \in \mathrm{PS}(S)$.

Definition 2.2 (Hyperoperation). (cf. [25, 26]) Let S be a nonempty set. A *(binary) hyperoperation* on S is a map

$$\circ : S \times S \longrightarrow \mathrm{PS}(S).$$

Thus, for $x, y \in S$, the value $x \circ y$ is a subset of S, not necessarily a single element.

Definition 2.3 (Hyperstructure). (cf. [26, 27, 1]) Let S be a nonempty set and let $\circ : S \times S \to \mathrm{PS}(S)$ be a hyperoperation. The pair

$$\mathcal{H} = (S, \circ)$$

is called a *hyperstructure* on S.

Extension to subsets. The hyperoperation $\circ$ extends canonically to a map

$$\circ : \mathrm{PS}(S) \times \mathrm{PS}(S) \longrightarrow \mathrm{PS}(S)$$

by the rule

$$A \circ B := \bigcup_{a \in A,\ b \in B} (a \circ b), \qquad A, B \subseteq S.$$

In this sense, a hyperstructure induces a natural "subset–level" composition on $\mathrm{PS}(S)$.

A *SuperHyperStructure* refines this idea by iterating the powerset construction. Instead of working only with $\mathrm{PS}(S)$, one allows operations whose inputs and outputs may lie in higher iterated powersets $\mathrm{PS}_n(S)$, so that operations can act on nested collections (sets of sets, sets of sets of sets, and so on). This provides a natural framework for multi–layer and hierarchical interactions [28, 29]. Typical examples include SuperHyperGraphs [15, 19, 30] and SuperHyperUncertain Sets [21].

Definition 2.4 (Iterated powersets). (cf. [1]) Let S be a nonempty set. Define recursively

$$\mathrm{PS}_0(S) := S, \qquad \mathrm{PS}_{n+1}(S) := \mathrm{PS}\big(\mathrm{PS}_n(S)\big) \quad (n \geq 0).$$

Thus $\mathrm{PS}_1(S) = \mathrm{PS}(S)$, $\mathrm{PS}_2(S) = \mathrm{PS}(\mathrm{PS}(S))$, etc.

The corresponding *nonempty* iterated powersets are defined by

$$\mathrm{PS}_0^*(S) := S, \qquad \mathrm{PS}_{n+1}^*(S) := \mathrm{PS}\big(\mathrm{PS}_n^*(S)\big) \setminus \{\emptyset\} \quad (n \geq 0).$$

Equivalently, for any set X, writing $\mathrm{PS}^*(X) := \mathrm{PS}(X) \setminus \{\emptyset\}$, we have $\mathrm{PS}_{n+1}^*(S) = \mathrm{PS}^*\big(\mathrm{PS}_n^*(S)\big)$.

Definition 2.5 $((m,n)$-SuperHyperOperation$)$. (cf. [1]) Let S be a nonempty set. Fix integers $m, n \geq 0$ and an arity $k \geq 1$. An (m,n)-*superhyperoperation of arity k* on S is a map

$$\star : \big(\mathrm{PS}_m(S)\big)^k \longrightarrow \mathrm{PS}_n(S).$$

If one wishes to exclude the empty set at each level, one may instead work with

$$\star : \big(\mathrm{PS}_m^*(S)\big)^k \longrightarrow \mathrm{PS}_n^*(S).$$

Definition 2.6 $((m,n)$-SuperHyperStructure$)$. (cf. [28, 31]) Let S be a nonempty set and fix integers $m, n \geq 0$ and an arity $k \geq 1$. A (m,n)-*SuperHyperStructure of arity k* on S is a pair

$$\mathcal{SH} = \big(S, \star\big),$$

where $\star : (\mathrm{PS}_m(S))^k \to \mathrm{PS}_n(S)$ is an (m,n)-superhyperoperation. Several familiar situations are recovered as special cases:

- if $m = n = 0$, then $\star : S^k \to S$ is an ordinary k-ary operation on S;

- if $m = 0$ and $n = 1$, then $\star : S^k \to \mathrm{PS}(S)$ is a k-ary hyperoperation;

- if $k = 1$, then $\star : \mathrm{PS}_m(S) \to \mathrm{PS}_n(S)$ is an (m,n)-superhyperfunction.

3   Main Results

3.1   Mixed HyperStructures and Mixed SuperHyperStructures

A *Mixed HyperStructure* couples several *typed* carrier sets and allows hyper-operations whose inputs and outputs may belong to different types, thereby capturing cross–type aggregation within a single formalism. A *Mixed Super-HyperStructure* extends this idea to *iterated* powersets, so that operations may act on nested collections and model hierarchical, multi–level interactions.

Definition 3.1 (Levels, canonical lifts, and reductions (depth ≤ 2)). Let S be a nonempty set. Define the first two iterated powersets

$$\mathrm{PS}_0(S) := S, \qquad \mathrm{PS}_1(S) := \mathrm{PS}(S),$$

$$\mathrm{PS}_2(S) := \mathrm{PS}\big(\mathrm{PS}(S)\big).$$

We use the canonical embeddings

$$\eta_S^{(0\to 1)} : \mathrm{PS}_0(S) \to \mathrm{PS}_1(S), \quad \eta_S^{(0\to 1)}(x) := \{x\},$$

$$\eta_S^{(1\to 2)} : \mathrm{PS}_1(S) \to \mathrm{PS}_2(S), \quad \eta_S^{(1\to 2)}(A) := \{A\}.$$

For later use we also fix the canonical *lifts* from level 1 to level $d \in \{1, 2\}$:

$$\mathrm{lift}_S^{(1\to 1)} := \mathrm{id}_{\mathrm{PS}_1(S)},$$

$$\mathrm{lift}_S^{(1\to 2)} := \eta_S^{(1\to 2)}.$$

Moreover, we define the canonical *reductions* to level 1 from $e \in \{0, 1, 2\}$ by

$$\mathrm{red}_S^{(0\to 1)} := \eta_S^{(0\to 1)}, \qquad \mathrm{red}_S^{(1\to 1)} := \mathrm{id}_{\mathrm{PS}_1(S)},$$

$$\mathrm{red}_S^{(2\to 1)} : \mathrm{PS}_2(S) \to \mathrm{PS}_1(S), \quad \mathrm{red}_S^{(2\to 1)}(\mathcal{F}) := \bigcup_{A \in \mathcal{F}} A.$$

Thus $\mathrm{red}_S^{(2\to 1)}$ forgets the internal grouping of subsets while retaining all elements that appear in at least one member of $\mathcal{F}$.

Remark. In general there is *no canonical* map $\mathrm{PS}_1(S) \to \mathrm{PS}_0(S)$ (that would require choosing an element of a subset), so we do not define such a reduction.

Definition 3.2 (Mixed HyperStructure). Let T be a nonempty set of *types*. For each $a \in T$ let S_a be a nonempty set, called the *base set of type a*. A *typed hyperprofile* is a pair

$$\lambda = \big((a_1, \ldots, a_k) \mid b\big) \in T^k \times T, \qquad k \geq 1.$$

A *Mixed HyperStructure* on the family $(S_a)_{a \in T}$ is a pair

$$\mathcal{H}_{\text{mix}} = \big((S_a)_{a \in T}, (\circ_\lambda)_{\lambda \in \Lambda} \big),$$

where Λ is an index set and, for each $\lambda = ((a_1, \ldots, a_k) \mid b) \in \Lambda$, the associated *mixed hyperoperation* is a map

$$\circ_\lambda : \text{PS}_1(S_{a_1}) \times \cdots \times \text{PS}_1(S_{a_k}) \longrightarrow \text{PS}_1(S_b).$$

No additional axioms are imposed unless explicitly stated.

We call $\mathcal{H}_{\text{mix}}$ *homogeneous* if T is a singleton, say $T = \{a\}$. Then $\circ_\lambda : (\text{PS}(S_a))^k \to \text{PS}(S_a)$ is a usual subset–level hyperoperation; restricting each input to a singleton $\{x_i\}$ yields an induced k-ary hyperoperation on S_a.

Definition 3.3 (Mixed SuperHyperStructure (depth ≤ 2)). Let T be a nonempty set of types and $(S_a)_{a \in T}$ a family of nonempty base sets. For each $a \in T$ we consider the levels $\text{PS}_d(S_a)$ for $d \in \{0, 1, 2\}$ and the canonical maps from Definition 3.1.

A *typed superhyperprofile of depth ≤ 2* is a datum

$$\mu = \big((a_1, d_1), \ldots, (a_k, d_k) \mid (b, e) \big),$$

where $k \geq 1$, $a_i, b \in T$, and $d_i, e \in \{0, 1, 2\}$. For such a profile define

$$\text{Dom}(\mu) := \text{PS}_{d_1}(S_{a_1}) \times \cdots \times \text{PS}_{d_k}(S_{a_k}), \qquad \text{Cod}(\mu) := \text{PS}_e(S_b).$$

A *Mixed SuperHyperStructure of depth ≤ 2* on $(S_a)_{a \in T}$ is a pair

$$\mathcal{SH}_{\text{mix}} = \big((S_a)_{a \in T}, (\star_\mu)_{\mu \in M} \big),$$

where M is an index set and, for each $\mu = ((a_1, d_1), \ldots, (a_k, d_k) \mid (b, e)) \in M$, the associated *mixed superhyperoperation* is a map

$$\star_\mu : \text{Dom}(\mu) \longrightarrow \text{Cod}(\mu).$$

PS_1**-compatible profiles and** PS_1**-reduction.** Call μ PS_1*-compatible* if $d_i \in \{1, 2\}$ for all i (so that each input can be lifted canonically from level 1 to level d_i). For such μ we define its PS_1*-reduction*

$$\star_\mu^{[1]} : \text{PS}_1(S_{a_1}) \times \cdots \times \text{PS}_1(S_{a_k}) \longrightarrow \text{PS}_1(S_b)$$

by

$$\star_\mu^{[1]}(A_1, \ldots, A_k) := \text{red}_{S_b}^{(e \to 1)}\Big(\star_\mu \big(\text{lift}_{S_{a_1}}^{(1 \to d_1)}(A_1), \ldots, \text{lift}_{S_{a_k}}^{(1 \to d_k)}(A_k) \big) \Big),$$

where $\mathrm{lift}_{S_{a_i}}^{(1 \to d_i)}$ and $\mathrm{red}_{S_b}^{(e \to 1)}$ are the canonical maps from Definition 3.1. In particular, for every PS$_1$-compatible μ, the map $\star_\mu^{[1]}$ is a mixed hyperoperation of type-only profile $\lambda(\mu) := ((a_1, \ldots, a_k) \mid b)$.

Hence, restricting to PS$_1$-compatible profiles yields a Mixed HyperStructure

$$\mathcal{H}_{\mathrm{mix}}^{(1)} := \big((S_a)_{a \in T}, \, (\star_\mu^{[1]})_{\mu \in M_{\mathrm{comp}}}\big),$$

where $M_{\mathrm{comp}} \subseteq M$ denotes the set of PS$_1$-compatible profiles.

Theorem 3.4 (Well-defined PS$_1$-reduction)**.** *Let $\mathcal{SH}_{\mathrm{mix}}$ be a Mixed SuperHyperStructure of depth ≤ 2 as in Definition 3.3. Fix a PS$_1$-compatible profile*

$$\mu = \big((a_1, d_1), \ldots, (a_k, d_k) \mid (b, e)\big) \in M_{\mathrm{comp}},$$

so that $d_i \in \{1, 2\}$ for all i and $e \in \{0, 1, 2\}$. Then the map $\star_\mu^{[1]}$ defined in Definition 3.3 is well-defined and satisfies

$$\star_\mu^{[1]} : \mathrm{PS}_1(S_{a_1}) \times \cdots \times \mathrm{PS}_1(S_{a_k}) \longrightarrow \mathrm{PS}_1(S_b).$$

Moreover, the family $(\star_\mu^{[1]})_{\mu \in M_{\mathrm{comp}}}$ forms a Mixed HyperStructure $\mathcal{H}_{\mathrm{mix}}^{(1)}$ on $(S_a)_{a \in T}$.

Proof. Let $A_i \in \mathrm{PS}_1(S_{a_i})$ be arbitrary. Since μ is PS$_1$-compatible, we have $d_i \in \{1, 2\}$, hence

$$X_i := \mathrm{lift}_{S_{a_i}}^{(1 \to d_i)}(A_i) \in \mathrm{PS}_{d_i}(S_{a_i}) \qquad (i = 1, \ldots, k)$$

by Definition 3.1. Therefore $(X_1, \ldots, X_k) \in \mathrm{Dom}(\mu)$ and the value

$$Y := \star_\mu(X_1, \ldots, X_k)$$

is well-defined and lies in $\mathrm{Cod}(\mu) = \mathrm{PS}_e(S_b)$. Applying the canonical reduction $\mathrm{red}_{S_b}^{(e \to 1)} : \mathrm{PS}_e(S_b) \to \mathrm{PS}_1(S_b)$ (from Definition 3.1) yields

$$\star_\mu^{[1]}(A_1, \ldots, A_k) = \mathrm{red}_{S_b}^{(e \to 1)}(Y) \in \mathrm{PS}_1(S_b).$$

Hence $\star_\mu^{[1]}$ is a well-defined map with the stated domain and codomain.

Finally, for each $\mu \in M_{\mathrm{comp}}$ the map $\star_\mu^{[1]}$ has signature

$$\mathrm{PS}_1(S_{a_1}) \times \cdots \times \mathrm{PS}_1(S_{a_k}) \to \mathrm{PS}_1(S_b),$$

which is exactly the signature of a mixed hyperoperation in Definition 3.2. Therefore the family $\mathcal{H}_{\mathrm{mix}}^{(1)} = \big((S_a)_{a \in T}, (\star_\mu^{[1]})_{\mu \in M_{\mathrm{comp}}}\big)$ is a Mixed HyperStructure. $\qquad \square$

Theorem 3.5 (Mixed SuperHyperStructures generalize SuperHyperStructures). *Fix integers $k \geq 1$ and $m, n \in \{0, 1, 2\}$, and let*

$$\mathcal{SH} = (S, \star)$$

be an (m, n)-SuperHyperStructure of arity k, i.e.,

$$\star : \left(\mathrm{PS}_m(S)\right)^k \longrightarrow \mathrm{PS}_n(S).$$

Then there exists a Mixed SuperHyperStructure of depth ≤ 2,

$$\mathcal{SH}_{\mathrm{mix}} = \left((S_a)_{a \in T}, (\star_\mu)_{\mu \in M}\right),$$

with a singleton type set $T = \{a\}$ and $S_a = S$, such that $\mathcal{SH}$ is recovered as the unique operation of $\mathcal{SH}_{\mathrm{mix}}$ on the homogeneous profile

$$\mu = \left((a, m), \ldots, (a, m) \mid (a, n)\right).$$

In particular, every (m, n)-SuperHyperStructure (with $m, n \leq 2$) is a special case of a Mixed SuperHyperStructure.

Conversely, if T is a singleton, then every Mixed SuperHyperStructure of depth ≤ 2 is canonically a family of (possibly many) superhyperoperations $\star_\mu : (\mathrm{PS}_d(S))^k \to \mathrm{PS}_e(S)$ on the same base set S, and hence contains ordinary (m, n)-SuperHyperStructures as homogeneous special cases.

Proof. Let $T = \{a\}$ and set $S_a := S$. Let $M := \{\mu\}$ with

$$\mu = \left((a, m), \ldots, (a, m) \mid (a, n)\right),$$

and define $\star_\mu := \star$.

By construction,

$$\mathrm{Dom}(\mu) = \underbrace{\mathrm{PS}_m(S_a) \times \cdots \times \mathrm{PS}_m(S_a)}_{k \text{ factors}} = (\mathrm{PS}_m(S))^k,$$

$$\mathrm{Cod}(\mu) = \mathrm{PS}_n(S_a) = \mathrm{PS}_n(S),$$

so $\star_\mu : \mathrm{Dom}(\mu) \to \mathrm{Cod}(\mu)$ is a well-defined mixed superhyperoperation. Hence $\mathcal{SH}_{\mathrm{mix}} = \left((S_a)_{a \in T}, (\star_\mu)_{\mu \in M}\right)$ is a Mixed SuperHyperStructure of depth ≤ 2, and under the identification $S_a = S$ we recover the original (m, n)-operation $\star$.

For the converse, if $T = \{a\}$ then all base sets coincide with $S := S_a$, and each profile $\mu = ((a, d_1), \ldots, (a, d_k) \mid (a, e))$ simply specifies a map $\star_\mu : \mathrm{PS}_{d_1}(S) \times \cdots \times \mathrm{PS}_{d_k}(S) \to \mathrm{PS}_e(S)$. Taking $d_1 = \cdots = d_k = m$ and $e = n$ yields an ordinary (m, n)-SuperHyperStructure as a homogeneous special case. $\square$

4 Open Problems

In this paper, we list several open problems.

- **Extension via Mixed SuperHyperStructures.** Can Mixed Super-HyperStructures genuinely extend classical hyperstructures and super-hyperstructures, and which minimal axioms (associativity/identity/inverses, if any) are appropriate?

- **Typed isomorphisms and invariants.** What is the right notion of type-preserving isomorphism, and which invariants classify Mixed (Super)HyperStructures up to such isomorphisms?

- PS_1**-reduction and realizability.** Which Mixed HyperStructures arise as PS_1-reductions of Mixed SuperHyperStructures?

- **Higher depth.** How should the definitions and reductions be extended to depth > 2, and what coherence conditions are needed for well-defined iterated reductions?

- **Examples and closure.** Construct nontrivial classes of examples and determine their closure under substructures, products, and quotients.

References

[1] Florentin Smarandache. Foundation of superhyperstructure & neutrosophic superhyperstructure. *Neutrosophic Sets and Systems*, 63(1):21, 2024.

[2] Sunday Adesina Adebisi and Adetunji Patience Ajuebishi. The order involving the neutrosophic hyperstructures, the construction and setting up of a typical neutrosophic group. *HyperSoft Set Methods in Engineering*, 3:26–31, 2025.

[3] M Al Tahan and Bijan Davvaz. Weak chemical hyperstructures associated to electro-chemical cells. *Iranian Journal of Mathematical Chemistry*, 9(1):65–75, 2018.

[4] GR Amiri, R Mousarezaei, and S Rahnama. Soft hyperstructures and their applications. *New Mathematics and Natural Computation*, pages 1–19, 2024.

[5] Alain Bretto. Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, 1, 2013.

[6] Yuxin Wang, Quan Gan, Xipeng Qiu, Xuanjing Huang, and David Wipf. From hypergraph energy functions to hypergraph neural networks. In *International Conference on Machine Learning*, pages 35605–35623. PMLR, 2023.

[7] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558–3565, 2019.

[8] Jayanta Ghosh and Tapas Kumar Samanta. Hyperfuzzy sets and hyperfuzzy group. *Int. J. Adv. Sci. Technol*, 41:27–37, 2012.

[9] Young Bae Jun, Kul Hur, and Kyoung Ja Lee. Hyperfuzzy subalgebras of bck/bci-algebras. *Annals of Fuzzy Mathematics and Informatics*, 2017.

[10] K Settu and M Jayalakshmi. Prediction of risk factor in hepatitis diagnosis using interval value hyperneutrosophic and einstein aggregated operations with extended mcdm. *Expert Systems with Applications*, page 130364, 2025.

[11] B Davvaz. A brief survey on algebraic hyperstructures: Theory and applications. *Journal of Algebraic Hyperstructures and Logical Algebras*, 1(3):15–29, 2020.

[12] Florentin Smarandache. Extension of hyperalgebra to superhyperalgebra and neutrosophic superhyperalgebra (revisited). In *International Conference on Computers Communications and Control*, pages 427–432. Springer, 2022.

[13] Hacı Mustafa PAKSOY, Mehmet Ali YÜZBAŞIOĞLU, and Yunus Emre. A theoretical framework of neutrosophic superhyperstructure in cost accounting: A new perspective for academic researchers. *SuperHyperTopologies and SuperHyperStructures with their Applications*, page 300, 2025.

[14] Florentin Smarandache, Giorgio Nordo, Saeid Jafari, et al. *SuperHyperTopologies and SuperHyperStructures with their Applications*. Biblio Publishing, 2025.

[15] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.

[16] Takaaki Fujita and Florentin Smarandache. *HyperGraph and SuperHyperGraph Theory with Applications*. Neutrosophic Science International Association (NSIA) Publishing House, 2026.

[17] Takaaki Fujita and Florentin Smarandache. *HyperGraph and SuperHyperGraph Theory with Applications (II): Graph Property and Parameter*, volume II of *HyperGraph and SuperHyperGraph Theory with Applications*. Neutrosophic Science International Association (NSIA) Publishing House, 1.0 edition, 2026.

[18] Takaaki Fujita and Florentin Smarandache. *HyperGraph and SuperHyperGraph Theory with Applications (IV): Uncertain Graph Theory*, volume IV of *HyperGraph and SuperHyperGraph Theory with Applications*. Neutrosophic Science International Association (NSIA) Publishing House, 1.0 edition, 2026.

[19] Masoud Ghods, Zahra Rostami, and Florentin Smarandache. Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties. *Neutrosophic Sets and Systems*, 50:480–487, 2022.

[20] Takaaki Fujita. Hyperfuzzy and superhyperfuzzy group decision-making. *Spectrum of Decision Making and Applications*, pages 1–18, 2027.

[21] Florentin Smarandache. *Hyperuncertain, superuncertain, and superhyperuncertain sets/logics/probabilities/statistics*. Infinite Study, 2017.

[22] Abdullah Kargın and Memet Şahin. Superhyper groups and neutro–superhyper groups. *2023 Neutrosophic SuperHyperAlgebra And New Types of Topologies*, 25, 2023.

[23] Florentin Smarandache. *Introduction to SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra*. Infinite Study, 2022.

[24] Thomas Jech. *Set theory: The third millennium edition, revised and expanded*. Springer, 2003.

[25] Souzana Vougioukli. Helix hyperoperation in teaching research. *Science & Philosophy*, 8(2):157–163, 2020.

[26] Piergiulio Corsini and Violeta Leoreanu. *Applications of hyperstructure theory*, volume 5. Springer Science & Business Media, 2013.

[27] Bijan Davvaz and Thomas Vougiouklis. *Walk Through Weak Hyperstructures, A: Hv-structures*. World Scientific, 2018.

[28] Florentin Smarandache. Superhyperstructure & neutrosophic superhyperstructure, 2024. Accessed: 2024-12-01.

[29] Ajoy Kanti Das, Rajat Das, Suman Das, Bijoy Krishna Debnath, Carlos Granados, Bimal Shil, and Rakhal Das. A comprehensive study of neutrosophic superhyper bci-semigroups and their algebraic significance. *Transactions on Fuzzy Sets and Systems*, 8(2):80, 2025.

[30] Mohammad Hamidi and Mohadeseh Taghinezhad. *Application of Superhypergraphs-Based Domination Number in Real World*. Infinite Study, 2023.

[31] Florentin Smarandache. *SuperHyperFunction, SuperHyperStructure, Neutrosophic SuperHyperFunction and Neutrosophic SuperHyperStructure: Current understanding and future directions*. Infinite Study, 2023.