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A Note on Homoderivations in Prime Ideals

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Abstract

Let R be a ring, P a prime ideal of R and $h : R \to R$ a homoderivation of R. In this paper, we proved that R/P is commutative integral domain in the ring R satisfying some algebraic identities.

Keywords: *prime ideal, homoderivation, commutativity.*
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1 Introduction

The derivations and their generalizations play major role in mathematics, economics, quantum physics and biology such as chemotheraphy. Many researchers have presented derivations of various algebraic structures such as rings, near rings. The commutativity of prime or semiprime rings with derivation was first discussed by E. C. Posner [6]. In this paper, Posner describes the definition of derivation in any ring as follows: An additive mapping d on R is a derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$ and proved that if a nonzero derivation d centralizng on a prime ring R, then R becomes commutative. Over the last several years, a number of authors studied commutativity theorems for prime or semiprime rings admitting automorphisms or derivations on appropriate subsets of R. Also, they investigated the commutativity using the concept of different derivation definitions. Because of that the correlation between derivations and the algebraic structures has become an exciting subject in the the last years.

Inspired by previous studies, the commutativity of rings has been discussed in more expansive way. In recent years, the effects of these conditions on the derivation of prime and semiprime ideals have begun to be examined. An ideal P in a ring R is said to be prime if $P \neq R$ and for any ideals A, B in R, $AB \subseteq P$ then $A \subseteq P$ or $B \subseteq P$. A ring R is said to be a prime ring if the zero ideal is a prime ideal (that is, if I, J are ideals such that $IJ = 0$, then $I = 0$ or $J = 0$). A ring R is semiprime if and only if R has no nonzero nilpotent ideals. Equivently, a ring R is said to be prime if $xRy = (0)$ implies that either $x = 0$ or $y = 0$ and semiprime if $xRx = (0)$.implies that $x = 0$, where $x, y \in R$. Every prime ring is semiprime since 0 is a prime ideal. Also, we know that R/P is a prime ring where P is prime ideal of R. The relationships among prime ideals, prime rings and semiprime rings are analogous to the relationships between mappings and commutativity. The consideration of whether the ring R is prime or semiprime has been omitted, and instead the focus has shifted to analyzing the beheviour of a factor ring R/P, where P is prime ideal of R.

In 2000, El Sofy [12] defined a homoderivation on R as an additive mapping $H : R \to R$ satisfying $H(xy) = H(x)H(y) + H(x)y + xH(y)$ for all $x, y \in R$. An example of such mapping is to let $H(x) = f(x) - x$, for all $x, y \in R$ where f is an endomorphism on R. It is clear that a homoderivation H is also a derivation if $H(x)H(y)$ =0 for all $x, y \in R$. In this case, $H(x)RH(y) = 0$ for all $x, y \in R$. Hence if R is a prime ring, then the only additive mapping which is both a derivation and a homoderivation is the zero mapping.

In [4], Daif and Bell proved that R is semiprime ring, I is a nonzero ideal of R and d is a derivation of R such that $d([x, y]) = \pm[x, y]$, for all $x, y \in I$, then R contains a nonzero central ideal. This theorem considered for homoderivations by El Sofy in [12]. Further, Hongan [5] extended this theorem as follows: Let R be a 2-torsion free semiprime ring and R a nonzero ideal of R and d a derivation of R. If $d([x, y]) \pm [x, y] \in Z$, for all $x, y \in R$, then $R \subseteq Z$.

There is a growing literature on strong commutativity preserving (SCP) maps and derivations. Bell and Daif [2] first investigated the derivation of SCP maps on the ideal of a semiprime ring. Bresar [3] generalized this work to the Lie ideal of the ring. In [7], Ma and Xu handled this study for generalized derivations. Moreover, Koç and Gölbaşı [8] have been studied for the multiplicative generalized derivations by generalizing these conditions on the semiprime ring. Koç Sögütcü addressed this condition for homoderivations in semiprime rings in 2023 [9].

The commutativity of prime and semiprime rings admitting derivations remains an active area of research. Recent approaches involve examining commutativity conditions in quotient rings rather than assuming the ring is prime (see, e.g. [1], [10], [11]).

From the above results, our aim is to explore a more general context of differential identities involving a prime ideal by omitting the primeness assumption imposed on the ring. This approach allows us to generalize the results obtained earlier.

2 Results

Throughout this paper, R will be a ring, P a prime ideal of R and h a homoderivation which is zero-power valued on R.

For any $x, y \in R$, as usual $[x, y] = xy - yx$ and $xoy = xy + yx$ will denote the well-known Lie and Jordan product, respectively and make extensive use of basic commutator identities:

$$\begin{aligned}[x, yz] &= y[x, z] + [x, y]z\\ [xy, z] &= [x, z]y + x[y, z]\\ xo(yz) &= (xoy)z - y[x, z] = y(xoz) + [x, y]z\\ (xy)oz &= x(yoz) - [x, z]y = (xoz)y + x[y, z].\end{aligned}$$

For all $x, y \in R$, we get

$$\begin{aligned}h([x, y]) &= h(xy - yx) = h(xy) - h(yx)\\ &= h(x)h(y) + h(x)y + xh(y) - h(y)h(x) - h(y)x - yh(x)\\ &= [h(x), h(y)] + [h(x), y] + [x, h(y)].\end{aligned}$$

Theorem 2.1 *Let R be a ring, P a prime ideal of R and h a homoderivation which is zero-power valued on R. If $[h(x), h(y)] \pm [x, y] \in P$ for all $x, y \in R$, then R/P is a commutative integral domain.*

Proof. By the hypothesis, we get

$$[h(x), h(y)] \pm [x, y] \in P \text{ for all } x, y \in R.$$

Replacing y by yz in the last expression, we have

$$\begin{aligned}&[h(x), h(y)]h(z) + h(y)[h(x), h(z)] + [h(x), h(y)]z + h(y)[h(x), z]\\ &+[h(x), y]h(z) + y[h(x), h(z)] \pm [x, y]z \pm y[x, z] \in P.\end{aligned}$$

Using the hypothesis, we get

$$[h(x), h(y)]h(z) + h(y)[h(x), h(z)] + +h(y)[h(x), z] + [h(x), y]h(z) \in P.$$

Again using our hypothesis, we can write the last expressino such as

$$\begin{aligned}&([x, y] + p_1)h(z) + h(y)([x, z] + p_2) + h(y)[h(x), z]\\ &+ [h(x), y]h(z) \in P, \text{ for any } p_1, p_2 \in P.\end{aligned}$$

That is

$$[x,y]h(z)+h(y)[x,z]+h(y)[h(x),z]+[h(x),y]h(z)\in P$$

and so

$$[x+h(x),y]h(z)+h(y)[h(x)+x,z]\in P.$$

Since h is zero-power valued on R, there exists an integer $n>1$ such that $h^n(x)=0$ for all $x\in R$. Replacing x by $x-h(x)+h^2(x)+...+(-1)^{n-1}h^{n-1}(x)$ in this expression, we get

$$[x,y]h(z)+h(y)[x,z]\in P.$$

Replacing z by x in the last expression, we get

$$[x,y]h(x)\in P \text{ for all } x,y\in R. \tag{1}$$

Replacing y by yz in the above expression, we get

$$[x,y]zh(x)\in P, \text{ for all } x,y,z\in R.$$

Since P is prime ideal, we have

$$[x,y]\in P \text{ or } h(x)\in P, \text{ for all } x,y\in R.$$

Let $L=\{x\in R\mid h(x)\in P\}$ and $K=\{x\in R\mid [x,y]\in P, \text{ for all } y\in R\}$. Clearly each of L and K is additive subgroup of R such that $R=L\cup K$. But, a group can not be the set-theoretic union of two proper subgroups. Hence $L=R$ or $K=R$. In the former case, $h(x)\in P$ for all $x\in R$. Using this in our hypothesis, we arrive at $[x,y]\in P$ for all $x,y\in R$. So we must have $[x,y]\in P$ for all $x,y\in R$ for any cases. Hence we get $xy-yx\in P$ for all $x,y\in R$. This implies that $(x+P)(y+P)=(y+P)(x+P)$ for all $x,y\in R$. We have R/P is a commutative integral domain. This completes the proof.

Theorem 2.2 *Let R be a ring, P a prime ideal of R and h a homoderivation which is zero-power valued on R.If any of the following expressions satisfies for all $x,y\in R$,*
i) $xh(y)+xy\in P$,
ii) $xh(y)+yx\in P$,
iii) $xh(y)\pm x\circ y\in P$,
iv) $[h(x),y]\pm xy\in P$,
v) $[h(x),y]\pm yx\in P$.
then R/P is a commutative integral domain.

Proof. i) By the hypothesis, we get

$$xh(y) + xy \in P, \text{ for all } x, y \in R.$$

That is,

$$x(h(y) + y) \in P, \text{ for all } x, y \in R.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing y by $y - h(y) + h^2(y) + ... + (-1)^{n-1}h^{n-1}(y)$ in this expression, we obtain that

$$xy \in P, \text{ for all } x, y \in R.$$

That is, $yx \in P$ for all $x, y \in R$. We conclude that $xy - yx \in P$ for all $x, y \in R$. We conclude that R/P is a commutative integral domain. We complete the proof.

ii) We get

$$xh(y) + yx \in P, \text{ for all } x, y \in R.$$

Replacing y by xy, $y \in R$ in this expression, we have

$$xh(x)h(y) + xh(x)y + x^2h(y) + xyx \in P.$$

Using the hypothesis, we arrive at

$$xh(x)(h(y) + y) \in P.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing y by $y - h(y) + h^2(y) + ... + (-1)^{n-1}h^{n-1}(y)$ in this expression, we obtain that

$$xh(x)y \in P, \text{ for all } x, y \in R.$$

Taking y by $rxh(x)$, $r \in R$ in the last expression, we have

$$xh(x)rxh(x) \in P, \text{ for all } x, y, r \in R.$$

By the primeness of P, we get

$$xh(x) \in P, \text{ for all } x, y \in R. \tag{2}$$

By the hypothesis, we get

$$xh(x) + x^2 \in P, \text{ for all } x \in R.$$

Using expression (2), we obtain that

$$x^2 \in P, \text{ for all } x \in R. \tag{3}$$

Replacing x by $x+y$ in this expression, we see that

$$x \circ y \in P, \text{ for all } x, y \in R. \tag{4}$$

Replacing y by yz, $z \in R$ in the above expression and using this, we get

$$[x, y]z \in P, \text{ for all } x, y, z \in R.$$

Replacing z by $z[x, y]$ in this expression, we have

$$[x, y]R[x, y] \in P, \text{ for all } x, y \in R.$$

Since P is prime ideal, we get

$$[x, y] \in P, \text{ for all } x, y \in R.$$

We conclude that R/P is a commutative integral domain. We complete the proof.

iii) By the hypothesis, we get

$$xh(y) \pm x \circ y \in P.$$

Replacing y by yx in this expression, we get

$$xh(y)h(x) + xh(y)x + xyh(x) \pm (x \circ y)x \in P.$$

Using the hypothesis, we get

$$xh(y)h(x) + xyh(x) \in P.$$

That is,

$$x(h(y) + y)h(x) \in P.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing y by $y - h(y) + h^2(y) + ... + (-1)^{n-1}h^{n-1}(y)$ in this expression, we obtain that

$$xyh(x) \in P, \text{ for all } x, y \in R.$$

Primenessly of P, we have

$$x \in P \text{ or } h(x) \in P, \text{ for all } x \in R.$$

Let $L = \{x \in R \mid x \in P\}$ and $K = \{x \in R \mid h(x) \in P\}$. Clearly each of L and K is additive subgroup of R such that $R = L \cup K$. But, a group can not be the set-theoretic union of two proper subgroups. Hence $L = R$ or $K = R$. In the former case, $x \in P$ for all $x \in R$, and so $P = R$.This contradicts that

P is prime ideal of R. So, we must have $K = R$. Hence we get $h(x) \in P$ for all $x \in R$. Using this in our hypothesis, we conclude that $x \circ y \in P$, for all $x, y \in R$. Appliying the same arguments after the equation (4), we get the required result.

iv) We get

$$[h(x), y] \pm xy \in P, \text{ for all } x, y \in R.$$

Taking y by yx in this expression, we get

$$[h(x), y]x + y[h(x), x] \pm xyx \in P.$$

By the hypothesis, we get

$$y[h(x), x] \in P, \text{ for all } x, y \in R. \tag{5}$$

Replacing y by $[h(x), x]r$, $r \in R$ in this eqaution, we get

$$[h(x), x]r[h(x), x] \in P.$$

Since P is prime ideal, we have

$$[h(x), x] \in P, \text{ for all } x \in R. \tag{6}$$

By the hypothesis, we get

$$[h(x), x] \pm x^2 \in P.$$

Using expression (6), we see that

$$x^2 \in P, \text{ for all } x \in R.$$

The rest of the proof is the same as expression (3). We complete the proof.

v) By the hypothesis, we get

$$[h(x), y] \pm yx \in P, \text{ for all } x, y \in R.$$

Replacing y by xy in this expression, we have

$$x[h(x), y] + [h(x), x]y \pm xyx \in P.$$

That is

$$[h(x), x]y \in P, \text{ for all } x, y \in R.$$

The rest of the proof is the same as expression (5). We complete the proof.

Theorem 2.3 *Let R be a ring, P a prime ideal of R and h a homoderivation which is zero-power valued on R.If any of the following expressions satisfies for all $x, y \in R$,*
i) $[h(x), y] \in P$,
ii) $h(x) \circ y \in P$,
iii) $h([x, y]) \pm [h(x), y] \in P$,
iv) $h(x \circ y) \pm h(x) \circ y \in P$
then R/P is a commutative integral domain.

Proof. i) By the hypothesis, we have

$$[h(x), y] \in P, \text{ for all } x, y \in R.$$

Replacing x by xz, $z \in R$ in the last expression, we get

$$[h(x)h(z) + h(x)z + xh(z), y] \in P, \text{ for all } x, y, z \in R.$$

Expanding this expression using our hypothesis, we find that

$$h(x)[z, y] + [x, y]h(z) \in P, \text{ for all } x, y, z \in R.$$

Taking z by y in the last expression, we have

$$[x, y]h(x) \in P, \text{ for all } x, y \in R.$$

The rest of the proof is the same as expression (1). We complete the proof.

ii) By the hypothesis, we get

$$h(x) \circ y \in P, \text{ for all } x, y \in R.$$

Replacing y by $yz, z \in R$ in this expression and using this expression, we have

$$y[z, h(x)] \in P, \text{ for all } x, y, z \in R.$$

Taking y by $[z, h(x)]r$, $r \in R$ in this expression, we get

$$[z, h(x)]r[z, h(x)] \in P, \text{ for all } x, z, r \in R.$$

That is,

$$[z, h(x)]R[z, h(x)] \subset P, \text{ for all } x, z \in R.$$

By the primeness of P, we find that

$$[z, h(x)] \in P, \text{ for all } x, z \in R.$$

By Theorem 3 (i), we conclude that R/P is a commutative integral domain.

iii) Let assume that

$$h([x,y]) \pm [h(x),y] \in P, \text{ for all } x,y \in R.$$

This implies that

$$[h(x),h(y)] + [h(x),y] + [x,h(y)] \pm [h(x),y] \in P$$

and so

$$[h(x),h(y)] + [x,h(y)] \in P, \text{ for all } x,y \in R.$$

That is

$$[h(x)+x,h(y)] \in P, \text{ for all } x,y \in R.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing x by $x - h(x) + h^2(x) + ... + (-1)^{n-1}h^{n-1}(x)$ in this expression, we get

$$[x,h(y)] \in P, \text{ for all } x,y \in R.$$

By Theorem 3 (i), we conclude that R/P is a commutative integral domain.

iv) We get

$$h(x \circ y) \pm h(x) \circ y \in P, \text{ for all } x,y \in R.$$

If this expression is edited, we have

$$h(x) \circ h(y) + h(x) \circ y + x \circ h(y) \pm h(x) \circ y \in P.$$

and so

$$h(x) \circ h(y) + x \circ h(y) \in P.$$

That is

$$(h(x)+x) \circ h(y) \in P, \text{ for all } x,y \in R.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing x by $x - h(x) + h^2(x) + ... + (-1)^{n-1}h^{n-1}(x)$ in this expression, we obtain that

$$x \circ h(y) \in P, \text{ for all } x,y \in R.$$

By Theorem 3 (ii), we get R/P is a commutative integral domain.

Theorem 2.4 *Let R be a ring, P a prime ideal of R and h a homoderivation which is zero-power valued on R.If any of the following expressions satisfies for all $x,y \in R$,*
i) $h(x)h(y) \pm xy \in P$, or
ii) $h(x)h(y) \pm yx \in P$.
Then R/P is a commutative integral domain or $h(R) \subset P$.

Proof. i) By the hypothesis, we get

$$h(x)h(y) \pm xy \in P \text{ for all } x, y \in R.$$

Replacing y by $yz, z \in R$ in this expression, we have

$$h(x)h(y)h(z) + h(x)h(y)z + h(x)yh(z) \pm xyz \in P.$$

Using the hypothesis, we get

$$h(x)h(y)h(z) + h(x)yh(z) \in P.$$

Again using our hypothesis, we can write the last expression

$$(xy + p_1)h(z) + h(x)yh(z) \in P, \text{ for any } p_1 \in P$$

and so

$$xyh(z) + h(x)yh(z) \in P. \tag{7}$$

Taking x by $rx, r \in R$ in the above expression, we find that

$$rxyh(z) + rh(x)yh(z) + h(r)h(x)yh(z) + h(r)xyh(z) \in P.$$

Using expression (7), we get

$$h(r)h(x)yh(z) + h(r)xyh(z) \in P.$$

That is

$$h(r)(h(x) + x)yh(z) \in P.$$

Since h is zero-power valued on R, there exists an integer $n > 1$ such that $h^n(x) = 0$ for all $x \in R$. Replacing x by $x - h(x) + h^2(x) + ... + (-1)^{n-1}h^{n-1}(x)$ in this expression, we get

$$h(r)xyh(z) \in P.$$

Replacing r by z, $z \in R$ in this expression, we get

$$h(z)xyh(z) \in P$$

and so

$$yh(z)Ryh(z) \subset P.$$

Since P is prime ideal, we have

$$yh(z) \in P, \text{ for all } y, z \in R.$$

Replacing y by $h(z)y$ in the last expression, we have

$$h(z)yh(z) \in P, \text{ for all } y, z \in R,$$

and so

$$h(z) \in P, \text{ for all } z \in R.$$

Using this in our hypothesis, we see that $xy \in P$, and so $[x, y] \in P$ for all $x, y \in R$. This implies that $xy - yx \in P$, and so $(x+P)(y+P) = (y+P)(x+P)$ for all $x, y \in R$. Hence we have R/P is a commutative integral domain. This completes proof.

ii) By the hypothesis, we have

$$h(x)h(y) \pm yx \in P, \text{ for all } x, y \in R.$$

Replacing y by $yz, z \in R$ in this expression, we get

$$h(x)h(y)h(z) + h(x)h(y)z + h(x)yh(z) \pm yzx \in P.$$

We can rewrite this expression such as

$$h(x)h(y)h(z) + h(x)h(y)z + h(x)yh(z) \pm yxz \mp yxz \pm yzx \in P.$$

Using the hypothesis, we obtain that

$$h(x)h(y)h(z) + h(x)yh(z) \mp yxz \pm yzx \in P.$$

Again using our hypothesis, we have

$$(yx + p_1)h(z) + h(x)yh(z) \mp yxz \pm yzx \in P.$$

This implies that

$$yxh(z) + h(x)yh(z) \mp yxz \pm yzx \in P. \tag{8}$$

Replacing y by $ry, r \in R$ in this expression, we obtain that

$$h(x)ryh(z) + ryxh(z) \mp ryxz \pm ryzx \in P.$$

Using the expression (8) and the hypothesis, we can rewrite the last equation such as

$$h(x)ryh(z) - r(h(x)yh(z) + p_2) \in P, \text{ for any } p_2 \in P$$

and so

$$h(x)ryh(z) - rh(x)yh(z) \in P.$$

Replacing r by $h(z)$ in the above expression, we find that

$$h(x)h(z)yh(z) - h(z)h(x)yh(z) \in P.$$

Using the hypothesis, we see that

$$(zx + p_3)yh(z) - (xz + p_4)yh(z), \text{ for any } p_3, p_4 \in P$$

and so

$$[x,z]yh(z) \in P, \text{ for all } x,y,z \in R.$$

The rest of the proof is the same as Theorem 1 (i). This completes proof.

Example. Suppose the ring $R = \left\{ \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \middle| a,b,c,d \in \mathbb{R} \right\}$. Define maps $h : R \to R$ as follows:

$$h\left(\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \right) = \left(\begin{array}{cc} 0 & b \\ 0 & 0 \end{array} \right).$$

Let

$$P = \left\{ \left(\begin{array}{cc} a & b \\ 0 & d \end{array} \right) \middle| a,b,d \in \mathbb{R} \right\}.$$

It is obvious that P is not a prime ideal. Then it is easy to verify that h is a homoderivation of R and $[h(x),y] \in P$ for all $x,y \in R$. However, R/P is not commutative.

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3 Open Problem

Our hypotheses are considered for the homoderivation on a prime ideal of the ring. Considering all hypotheses on the ring with semiprime ideals gives more general results. Also, if different derivations and conditions are considered on prime or semiprime ideals of the ring, many papers on this topic turn out to have different results.

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