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Product operations on square soft-rough fuzzy matrices

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Abstract

Soft set theory is one of the recent generalizations to deal with uncertainty concepts. Soft-rough sets emerged on the fusion of soft sets and rough sets. Recently, square soft-rough matrices were introduced as an extension of soft-rough matrices. This study further expands them into square soft-rough fuzzy matrices (s-s-r fm) and explores some of their key properties.

Keywords: *Soft-rough sets, soft-rough matrices, square soft-rough matrices, square soft-rough fuzzy matrices.*
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1 Introduction

Uncertainty is tackled using several tools starting from probability theory to the recent innovative idea namely soft sets. Pawlak's rough set (r-s-t) theory [9] defines uncertainty using boundary regions instead of membership functions. Vijayabalaji and Balaji [14] developed rough matrices based on rough membership functions and proposed a decision-making framework. Molodtsov [7] highlighted the limitations of existing models, including rough sets and introduced soft sets (s-s-t) as a mathematical tool to improve uncertainty representation. These sets enable flexible descriptions using elements like real numbers, functions, and linguistic variables. Maji and Roy [5] applied soft sets to decision-making, while Wille [15] represented property systems in a binary structure. Several researchers [3, 9, 11, 12, 17] investigated these systems and their role in data analysis.

Cagman [1, 2] introduced a decision-making approach using soft matrices, demonstrating their efficiency in representing s-s-t for computational storage and processing. Vijayabalaji and Ramesh [13] explored product soft matrices and their application in decision-making. Feng Feng [4, 5] studied hybrid models combining r-s-t and s-s-t, leading to soft-rough sets (s-r-s), which redefine uncertainty handling using soft approximations instead of Pawlak's rough set boundaries.

Vijayabalaji [15] introduced the idea of soft-rough matrices (s-s-r -m) by assigning three values corresponding to the three regions notified in soft-rough sets (s-r-s). These matrices offer an improved method for analyzing uncertainty. He then recently generalized it to square soft-rough matrices [15]. Generalized s-s-r-m can be viewed in [8]. In the present paper, section 2 discusses the need for square soft-rough fuzzy matrices, while Section 3 introduces these matrices along with essential concepts and operational principles. Section 4 introduces two types of product operations on square soft-rough fuzzy matrices namely $\wedge$ and $\vee$ operations. The union and intersection of these matrices are explained clearly for better comprehension. Furthermore, essential theorems related to these product operations are presented, highlighting their mathematical significance.

2 Need of square soft-rough fuzzy matrices

This study expands the concept further by introducing square soft-rough fuzzy matrices (s-s-r f-m) and analyzes their key properties. A square soft-rough fuzzy matrix is structured (s-s-r f-m) as an $n \times n$ matrix. The primary advantage of using this format lies in its symmetry and uniformity, which simplify various mathematical operations and analyses. Because of its balanced structure, computations become more efficient, and the relationships between data elements are easier to interpret.

Furthermore, the structured nature of square soft-rough matrices enhances their usefulness in mathematical and computational models. Their adaptability allows them to be applied in different fields, offering a more systematic approach in handling uncertainty and decision-making processes.

2.1 Advantages of square soft-rough fuzzy matrices over square soft-rough matrices

Square soft-rough fuzzy matrices offer an advanced approach by incorporating fuzzy principles into square soft-rough matrices. While square soft-rough matrices maintain a structured $n \times n$ format for balanced computations, they rely on crisp classifications, which may not fully capture the uncertainty involved. By introducing fuzzy membership values, square soft-rough fuzzy matrices provide a more refined method to represent imprecise data, allowing for a smoother transition between categories. This flexibility enhances decision-making in situations where rigid classifications may lead to information loss.

Additionally, the integration of fuzziness improves computational stability by reducing abrupt shifts in classifications. This makes square soft-rough fuzzy matrices more suitable for handling complex datasets, such as those in medical diagnosis, financial analysis, and pattern recognition. Their adaptability ensures better results when working with dynamic information, making them a more effective tool for uncertainty management in mathematical and computational applications.

Throughout this paper let $\grave{U}$, $\grave{E}$, $\grave{A}$ $P(\grave{U})$denote the universe set, parameters set, subset of a parameter set and power set of the universe set, respectively.

Definition 2.1 [18]. A fuzzy set is defined as a membership function from $\grave{U} \to [0, 1]$.

Definition 2.2 [10]. Consider a binary relation $R \subseteq \grave{U} \times \grave{U}$. R is called as indiscernibility relation. Assuming R as an equivalence relation the pair $(\grave{U}, R)$ is called a rough approximation space. Let $\grave{X} \subseteq \grave{U}$, $R(x)$ denotes the equivalence class of R and is determined by the elements of $\grave{X}$. We define two approximations as follows.
$\underline{R}_*(\grave{X}) = \{x \in \grave{U} : R(x) \subseteq \grave{X}\}$,

$\overline{R}_*(\grave{X}) = \{x \in \grave{U} : R(x) \cap \grave{X} \neq \emptyset\}$,

$BN_R(\grave{X}) = \overline{R}_*(\grave{X}) - \underline{R}_*(\grave{X})$ is called the boundary region of $\grave{X}$.
A set $\grave{X}$ is called crisp with respect to the binary relation R if and only if the boundary region of $\grave{X}$ is empty. A set $\grave{X}$ is called rough with respect to the binary relation R if and only if the boundary region of $\grave{X}$ is non-empty.

Definition 2.3 [7]. A s-s-t is defined as a mapping from $\grave{A} \to P(\grave{U})$.

Definition 2.4 [5]. For a given soft set $\Im = (F, A)$ over $\grave{U}$ with the soft approximation space $P = (\grave{U}, \Im)$, two operations are presented as follows.
$\underline{apr}_P(\grave{X}) = \{u \in \grave{U} : \exists a \in A, (u \in f(a), f(a) \subseteq \grave{X})\}$,

$\overline{apr}_P(\grave{X}) = \{u \in \grave{U} : \exists a \in A, (u \in f(a), f(a) \cap \grave{X} \neq \phi)\}$,
assigning to every subset $\grave{X} \subseteq \grave{U}$ two sets $\underline{apr}_P(\grave{X})$ and $\overline{apr}_P(\grave{X})$ termed as lower and upper soft rough approximations of $\grave{X}$ in P, respectively. In addition,
$PTV_P(\grave{X}) = \underline{apr}_P(\grave{X})$, $NTV_P(\grave{X}) = \grave{U} - \overline{apr}_P(\grave{X})$, $BDRY_P(\grave{X}) = \overline{apr}_P(\grave{X}) - \underline{apr}_P(\grave{X})$ are called the soft positive, soft negative and soft boundary regions of X, respectively. If $\underline{apr}_P(\grave{X}) = \overline{apr}_P(\grave{X})$, $\grave{X}$ is said to be soft definable; otherwise $\grave{X}$ is called a soft rough set.

Definition 2.5 [15]. A function defined by $C_{SR} : \grave{U} \to \{0, 0.5, 1\}$ over s-r-s is termed as s-s-r m.

Definition 2.6 [16]. A function defined by $I_{SR} : \grave{U} \to \{0, 0.5, 1\}$ over s-r-s is termed as s-s-r m.

3 Square soft-rough fuzzy matrices

This section presents the concept of square soft-rough fuzzy matrices with detailed explanations, along with essential definitions and theorems to enhance clarity and understanding.

Definition 3.1. A fuzzy set $I_{SR} : \grave{U} \to [0, 1]\}$ over s-r-s is termed as square soft-rough fuzzy matrix (s-s-r f-m) and denoted by ϖ_{SRF} . In particular ϖ_{SRF} takes the value 1, if $u \in PTV_p(\grave{X})$, 0 if $u \in NTV_p(\grave{X})$ and ζ if $u \in BDRY_p(\grave{X})$, with $\zeta \in (0, 1)$.

So, $\varpi_{SRF} = (\grave{\varrho}_{xy})_{nn}$.

Example 3.2. Let $\grave{U} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6\}$ and $\grave{E} = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6\}$. Let $\Im = (F, E)$ be a soft set over $\grave{U}$ with the following table:

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6
ε_1	1	0	0	1	0	0
ε_2	0	0	0	1	0	0
ε_3	0	1	0	0	0	0
ε_4	0	0	0	0	0	0
ε_5	0	0	0	1	0	0
ε_6	1	0	0	0	0	0

For $\grave{X} = \{\varepsilon_3, \varepsilon_4, \varepsilon_5\} \subset \grave{U}$, we have $\underline{apr}_P(\grave{X}) = \{\varepsilon_3\}$ and $\overline{apr}_P(\grave{X}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_5\}$.

Further $PTV_P(\grave{X}) = \{\varepsilon_3\}$, $NTV_P(\grave{X}) = \{\varepsilon_4, \varepsilon_6\}$ and $BDRY_P(\grave{X}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_5\}$.

Define a fuzzy set from $\varpi_{SRF1} : \grave{U} \to [0,1]$. Then our desired s-s-r fm is,

$\varpi_{SRF1} =$

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6
ε_1	ζ	ζ	ζ	ζ	ζ	ζ
ε_2	ζ	ζ	ζ	ζ	ζ	ζ
ε_3	1	1	1	1	1	1
ε_4	0	0	0	0	0	0
ε_5	ζ	ζ	ζ	ζ	ζ	ζ
ε_6	0	0	0	0	0	0

The elements are placed as per Definition 3.1.

Example 3.3. Let $\Im = (F, A)$ be a soft set over U as in Example 3.2 with the same table.

Choose $\grave{X} = \{\varepsilon_1\varepsilon_2, \varepsilon_3\} \subset \grave{U}$. Then we have $\underline{apr}_P(\grave{X}) = \{\varepsilon_3\}$ and $\overline{apr}_P(\grave{X}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_5, \varepsilon_6\}$

Further $PTV_P(\grave{X}) = \{\varepsilon_3\}$, $NTV_P(\grave{X}) = \{\varepsilon_4\}$ and $BDRY_P(\grave{X}) = \{\varepsilon_1, \varepsilon_2, \varepsilon_5, \varepsilon_6\}$.

Define a fuzzy set from $\varpi_{SRF2} : \grave{U} \to [0,1]$. Then our desired s-s-r f-m is,

$\varpi_{SRF2} =$

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6
ε_1	ζ	ζ	ζ	ζ	ζ	ζ
ε_2	ζ	ζ	ζ	ζ	ζ	ζ
ε_3	1	1	1	1	1	1
ε_4	0	0	0	0	0	0
ε_5	ζ	ζ	ζ	ζ	ζ	ζ
ε_6	ζ	ζ	ζ	ζ	ζ	ζ

Definition 3.4.
(i) Consider two soft-rough fuzzy matrices namely $[\varrho_{xy}]$ and $[\varsigma_{xy}] \in \varpi_{SRF}$. Then their intersection is defined by $[\varrho_{xy}] \cap [\varsigma_{xy}] = \min\{\varrho_{xy}, \varsigma_{xy}\}$.
(ii) Consider two soft-rough fuzzy matrices namely $[\varrho_{xy}]$ and $[\varsigma_{xy}] \in \varpi_{SRF}$. Then their union is defined by $[\varrho_{xy}] \cup [\varsigma_{xy}] = \max\{\varrho_{xy}, \varsigma_{xy}\}$.

Example 3.5. Consider the above Examples 3.2 and 3.3. Then

$\varpi_{SRF1} \cap \varpi_{SRF2}$
$= [\varrho_{xy}] \cap [\varsigma_{xy}]$
$= \min\{\varrho_{xy}, \varsigma_{xy}\}$

=

	e_1	e_2	e_3	e_4	e_5	e_6
u_1	ζ	ζ	ζ	ζ	ζ	ζ
u_2	ζ	ζ	ζ	ζ	ζ	ζ
u_3	1	1	1	1	1	1
u_4	0	0	0	0	0	0
u_5	ζ	ζ	ζ	ζ	ζ	ζ
u_6	0	0	0	0	0	0

and
$\varpi_{SRF1} \cup \varpi_{SRF2}$
$= [\varrho_{xy}] \cup [\varsigma_{xy}]$
$=\ \min\{\varrho_{xy}, \varsigma_{xy}\}$

	e_1	e_2	e_3	e_4	e_5	e_6
u_1	ζ	ζ	ζ	ζ	ζ	ζ
u_2	ζ	ζ	ζ	ζ	ζ	ζ
u_3	1	1	1	1	1	1
u_4	0	0	0	0	0	0
u_5	ζ	ζ	ζ	ζ	ζ	ζ
u_6	ζ	ζ	ζ	ζ	ζ	ζ

4 Product opertions on square soft-rough fuzzy matrices

In this section two product operations on square soft-rough fuzzy matrices, namely $\wedge$ and $\vee$ operations, are introduced. Their union and intersection are then defined clearly for better understanding. Additionally, key theorems related to these product operations are presented, highlighting their mathematical significance.

Definition 4.1. Consider two soft-rough fuzzy matrices namely $[(\grave{\varrho}_{xy}]$ and $[\grave{\varsigma}_{xy}] \in \grave{\varpi}_{SRF}$. Then the $\wedge$- product operations of $[(\grave{\varrho}_{xy}]$ and $[\grave{\varsigma}_{xy}]$ is defined by $\wedge : [\grave{\varrho}_{xy}] \times [\grave{\varsigma}_{yz}] \to [\grave{\varpi}_{xz}]^2$, termed as $\wedge\grave{\varpi}_{xz}$- square soft - rough fuzzy matrices or simply $\wedge\grave{\varpi}_{xz}$, where $\grave{\varpi}_{xz}$ =min$\{\grave{\varrho}_{xy}, \grave{\varsigma}_{yz}\}$.

Definition 4.2. Consider two soft-rough fuzzy matrices namely $[\grave{\varrho}_{xy}]$ and $[\grave{\varsigma}_{xy}] \in \grave{\varpi}_{SRF}$. Then the $\vee$- product operations of $[\grave{\varrho}_{xy}]$ and $[\grave{\varsigma}_{xy}]$ is defined by $\vee : [\grave{\varrho}_{xy}] \times [\grave{\varsigma}_{yz}] \to [\grave{\varpi}_{xz}]^2$, termed as $\vee\grave{\varpi}_{xz}$- square soft - rough fuzzy matrices or simply $\vee\grave{\varpi}_{xz}$,where $\grave{\varpi}_{xz}$ =max$\{\ \grave{\varrho}_{xy}, \grave{\varsigma}_{yz}\}$.

Definition 4.3. Consider two $\wedge\varpi_{xz}$- square soft-rough fuzzy matrices namely $\wedge_1\varpi_{xz}$ and $\wedge_2\varpi_{xz}$. Then
(i) $\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}$= max$\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}$
(i) $\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz}$= min$\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}$.

Definition 4.4. Consider two $\vee\varpi_{xz}$- square soft-rough fuzzy matrices namely $\vee_1\varpi_{xz}$ and $\vee_2\varpi_{xz}$. Then
(i) $\vee_1\varpi_{xz} \cup \vee_2\varpi_{xz}$= $\max\{\vee_1\varpi_{xz}, \vee_2\varpi_{xz}\}$
(i) $\vee_1\varpi_{xz} \cap \vee_2\varpi_{xz}$= $\min\{\vee_1\varpi_{xz}, \vee_2\varpi_{xz}\}$.

Theorem 4.5. Consider two $\wedge\varpi_{xz}$- square soft-rough fuzzy matrices namely $\wedge_1\varpi_{xz}$ and $\wedge_2\varpi_{xz}$. Then
(i) $\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz} = \wedge_2\varpi_{xz} \cup \wedge_1\varpi_{xz}$
(ii) $\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz} = \wedge_2\varpi_{xz} \cap \wedge_1\varpi_{xz}$
(iii) $\vee_1\varpi_{xz} \cup \vee_2\varpi_{xz} = \vee_2\varpi_{xz} \cup \vee_1\varpi_{xz}$
(iv) $\vee_1\varpi_{xz} \cap \vee_2\varpi_{xz} = \vee_2\varpi_{xz} \cap \vee_1\varpi_{xz}$
Proof.
(i) Consider
$\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}$
$=\max\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}$
$=\max\{\wedge_2\varpi_{xz}, \wedge_1\varpi_{xz}\}$
$=\wedge_2\varpi_{xz} \cup \wedge_1\varpi_{xz}$.
(ii) Also
$\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz}$
$=\min\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}$
$=\min\{\wedge_2\varpi_{xz}, \wedge_1\varpi_{xz}\}$
$=\wedge_2\varpi_{xz} \cap \wedge_1\varpi_{xz}$.
(iii) Now
$\vee_1\varpi_{xz} \cap \vee_2\varpi_{xz}$
$=\max\{\vee_1\varpi_{xz}, \vee_2\varpi_{xz}\}$
$=\max\{\vee_2\varpi_{xz}, \vee_1\varpi_{xz}\}$
$=\vee_2\varpi_{xz} \cap \vee_1\varpi_{xz}$.
(iv) Also
$\vee_1\varpi_{xz} \cap \vee_2\varpi_{xz}$
$=\min\{\vee_1\varpi_{xz}, \vee_2\varpi_{xz}\}$
$=\min\{\vee_2\varpi_{xz}, \vee_1\varpi_{xz}\}$
$=\vee_2\varpi_{xz} \cap \vee_1\varpi_{xz}$.

Theorem 4.6. Consider three $\wedge\varpi_{xz}$- square soft-rough fuzzy matrices namely $\wedge_1\varpi_{xz}$, $\wedge_2\varpi_{xz}$ and $\wedge_3\varpi_{xz}$ respectively. Then
(i) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cup \wedge_3\varpi_{xz} = \wedge_1\varpi_{xz} \cup (\wedge_2\varpi_{xz} \cup \wedge_3\varpi_{xz})$
(ii) $(\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz} = \wedge_1\varpi_{xz}(\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$
(iii) $(\vee_1\varpi_{xz} \cup \vee_2\varpi_{xz}) \cup \vee_3\varpi_{xz} = \vee_1\varpi_{xz}(\vee_2\varpi_{xz} \cup \vee_3\varpi_{xz})$
(iv) $(\vee_1\varpi_{xz} \cap \vee_2\varpi_{xz}) \cap \vee_3\varpi_{xz} = \vee_1\varpi_{xz}(\vee_2\varpi_{xz} \cap \vee_3\varpi_{xz})$
Proof.

(*i*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cup \wedge_3\varpi_{xz}$
$= \max\ \{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\} \cup \wedge_3\varpi_{xz}$
$= \max\ \{\max\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}, \wedge_3\varpi_{xz}\}$
$= \max\ \{\wedge_1\varpi_{xz}, \max\{\wedge_2\varpi_{xz}, \wedge_3\varpi_{xz}\}\}$
$= \wedge_1\varpi_{xz} \cup \max\{(\wedge_2\varpi_{xz} \cup \wedge_3\varpi_{xz})\}$
$= \wedge_1\varpi_{xz} \cup (\wedge_2\varpi_{xz} \cup \wedge_3\varpi_{xz})$.
ii) $(\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz}$
$= \min\ \{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\} \cap \wedge_3\varpi_{xz}$
$= \min\ \{\max\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}, \wedge_3\varpi_{xz}\}$
$= \min\ \{\wedge_1\varpi_{xz}, \max\{\wedge_2\varpi_{xz}, \wedge_3\varpi_{xz}\}\}$
$= \wedge_1\varpi_{xz} \cap \max\{(\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})\}$
$= \wedge_1\varpi_{xz} \cap (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$.

Similarly we can prove (*iii*) and (*iv*).

Theorem 4.7. Consider three $\wedge\varpi_{xz}$- square soft-rough fuzzy matrices namely $\wedge_1\varpi_{xz}$, $\wedge_2\varpi_{xz}$ and $\wedge_3\varpi_{xz}$ respectively. Then
(*i*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz} = (\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}) \cup (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$
(*ii*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz} = (\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}) \cup (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$
(*iii*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz} = (\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}) \cup (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$
(*iv*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz} = (\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}) \cup (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$
Proof.
(*i*) $(\wedge_1\varpi_{xz} \cup \wedge_2\varpi_{xz}) \cap \wedge_3\varpi_{xz}$
$= \max\ \{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\} \cap \wedge_3\varpi_{xz}$
$= \min\ \{\max\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}, \wedge_3\varpi_{xz}\}$
$= \max\ \{\min\{\wedge_1\varpi_{xz}, \wedge_3\varpi_{xz}\}, \min\{\wedge_2\varpi_{xz}, \wedge_3\varpi_{xz}\}\}$
$=\max\{\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}, \wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz}\}$
$= (\wedge_1\varpi_{xz} \cap \wedge_3\varpi_{xz}) \cup (\wedge_2\varpi_{xz} \cap \wedge_3\varpi_{xz})$.

(*ii*) $(\wedge_1\varpi_{xz} \cap \wedge_2\varpi_{xz}) \cup \wedge_3\varpi_{xz}$
$= \min\ \{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\} \cup \wedge_3\varpi_{xz}$
$= \max\ \{\min\{\wedge_1\varpi_{xz}, \wedge_2\varpi_{xz}\}, \wedge_3\varpi_{xz}\}$
$= \min\ \{\max\{\wedge_1\varpi_{xz}, \wedge_3\varpi_{xz}\}, \max\{\wedge_2\varpi_{xz}, \wedge_3\varpi_{xz}\}\}$
$=\min\{\wedge_1\varpi_{xz} \cup \wedge_3\varpi_{xz}, \wedge_2\varpi_{xz} \cup \wedge_3\varpi_{xz}\}$
$= (\wedge_1\varpi_{xz} \cup \wedge_3\varpi_{xz}) \cap (\wedge_2\varpi_{xz} \cup \wedge_3\varpi_{xz})$.

Similarly we can prove (*iii*) and (*iv*).

5 Conclusion

This paper explores square soft-rough fuzzy matrices (s-s-r f-m) and presents key theorems and results associated with them. It also examines joint and meet operations, providing insights into their mathematical properties and applications.

6 Open Problem

This research opens several avenues for further exploration.
(1) Can the inverse of s-s-r f-m be systematically determined.
(2) How can the determinant and adjoint of s-s-r f-m be defined and analyzed in a structured manner.
(3) Is there a way to develop efficient computational techniques for operations on s-s-r f-m.
(4) How does different types of fuzzy and rough approximations impact the structure and properties of s-s-r f-m.

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